

Joint Geometry/Probability Theory Seminar  
May 6, 2008, Tuesday, 6:00 p.m.  
Room 613, Courant Institute  
251 Mercer Street, New York

## Word maps and spectra of random graph lifts

Doron Puder  
New York and Jerusalem

### Abstract

We begin with a new analysis of formal words. Let  $w$  be a formal word in letters  $g_1, \dots, g_k$ . The word map associated with  $w$  maps the permutations  $s_1, \dots, s_k$  in  $S_n$  to the permutation obtained by replacing, for each  $i$ , every occurrence of  $g_i$  in  $w$  by  $s_i$ . We investigate the random variable  $X_{w,n}$  that counts the fixed points in this permutation when the  $s_i$  are selected uniformly at random. A major ingredient of our work is a new categorization of words which considerably extends the dichotomy of primitive vs. imprimitive words.

This analysis contributes deeply to our study of the spectra of random lifts of graphs. Let  $G$  be a connected graph, and let the infinite tree  $T$  be its universal cover space. We introduce an improved upper bound to all "new" eigenvalues of almost every  $n$ -lift of  $G$  in terms of the spectral radii of  $G$  and  $T$ . As an aside, we obtain a new conceptual and relatively simple proof of a theorem by A. Nica, which determines, for every fixed  $w$ , the limit distribution of  $X_{w,n}$ . A surprising aspect of this theorem is that the answer depends only on the largest integer  $d$  so that  $w = u^d$  for some word  $u$ . Joint work with Nati Linial.

For further information contact {pach,pollack}@cims.nyu.edu, or visit our website: [http://www.math.nyu.edu/seminars/geometry\\_seminar.html](http://www.math.nyu.edu/seminars/geometry_seminar.html)