Combinatorial complexity in $\omega$-minimal geometry

Saugata Basu
Georgia Tech

Abstract

It is customary in discrete and computational geometry to study the complexity of arrangements whose objects are of constant description complexity – where the phrase “constant description complexity” refers to the fact that the objects are semi-algebraic sets defined using a bounded number of polynomial equalities and inequalities of bounded degrees. In this talk I will present a much more general framework to study arrangements. The objects of our arrangements belong to some fixed definable family of sets in an $\omega$-minimal structure. Given an arrangement $A$ of $n$ definable subsets of $\mathbb{R}^k$ belonging to such a family, we show that the combinatorial and topological complexity of $A$ is bounded by $Cn^k$ where $C$ is a constant depending only on the family. We prove upper bounds on the size of a definable cylindrical decomposition of $\mathbb{R}^k$ compatible with $A$. Moreover, if $A$ belongs to a parametrized definable family of arrangements, we give a single exponential upper bound on the number of homotopy types of such arrangements, generalizing similar results obtained by Vorobjov and the author in the semi-algebraic and semi-Pfaffian case. Finally, as a sample application, I will describe an extension of a Ramsey-type theorem originally proved for semi-algebraic sets of fixed description complexity to this more general setting.

For further information contact \{pach,pollack\}@cims.nyu.edu, or visit our website: http://www.math.nyu.edu/seminars/geometry_seminar.html