Geometry Seminar
March 6, 2007, Tuesday, 6:00 p.m.
Room 613, Courant Institute
251 Mercer Street, New York

L_1-embeddings of the transportation cost metric on planar grids

Assaf Naor
Courant Institute, NYU

Abstract

Let (X, d) be a finite metric space. Given two k-point subsets of X, A and B, define the minimum cost matching between them as the minimum total displacement of a bijection between A and B, i.e. the minimum of $\sum_{a \in A} d(a, f(a))$ over all bijections $f : A \to B$. This distance is a particular case of the transportation cost metric on the set of probability measures on X. When X is the $n \times n$ planar grid, this metric has been long used in image processing as a measurement of similarity between images. The search for efficient nearest neighbor search data structures for this space of images led to the question whether the space of all probability measures on the $n \times n$ planar grid embeds into L_1 with distortion uniformly bounded in n. In this talk we will show that the L_1 distortion of this space is at least $c\sqrt{\log n}$, where c is a universal constant. We also describe an embedding into L_1 which achieves distortion $O(\log n)$. The proofs are based on methods from Banach space theory and harmonic analysis.

Joint work with Gideon Schechtman.

For further information contact {pach,pollack}@cims.nyu.edu, or visit our website: http://www.math.nyu.edu/seminars/geometry_seminar.html