Geometry Seminar
February 26, 2002, Tuesday, 6:00 p.m.
Room 1013, Courant Institute
251 Mercer Street, New York

Lower envelopes and their overlays

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Abstract

Lower envelopes of multivariate functions are a basic construct in many combinatorial and algorithmic problems in computational geometry. It is known that the combinatorial complexity of the lower envelope of n d-variate functions having 'constant description complexity' is $O(n^{d+\epsilon})$, for any $\epsilon > 0$. The proof, which is about eight years old, is rather involved, and is based on a 'charging scheme', in which vertices of the envelope are charged to other 'shallow' vertices of the arrangement of the function graphs.

The charging technique has been refined and extended to analyze the complexity of many other substructures in arrangements. One such extension is the overlay of the minimization diagrams of lower envelopes of two collections of bivariate functions; the same near-quadratic bound holds for the complexity of such an overlay.

In this talk we present a new and simpler analysis that uses a natural divide and conquer technique. We exemplify it to derive the known bounds for the complexity of lower envelopes of bivariate functions, and of the overlay of the minimization diagrams of two such envelopes.

However, the main new result is a near-cubic bound for the complexity of the overlay of the minimization diagrams of lower envelopes of trivariate functions. This has been a major open problem, and its solution leads to many interesting applications, such as a near-cubic bound for the complexity of the space of all line transversals to a set of compact convex sets in 3-space. (Joint work with Vladlen Koltun.)

For further information contact Janos Pach at pachcims.nyu.edu, or visit our website: http://www.math.nyu.edu/seminars/geometry_seminar.html