Geometry Seminar
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Room 1013, Courant Institute
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Maxwell's correspondence and gain graphs

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Abstract

In the 1860s Maxwell described a relationship between equilibrium stresses in a plane framework and polyhedral surfaces projected on this framework. In the simplest case, Maxwell's correspondence can also be interpreted in terms of lifting a tiling of the plane to a spatial surface. Lifting a tiling of R^d to a convex surface, tangent to a paraboloid, appears to be a useful technique in geometry of numbers (Voronoi, 1908) and computational geometry (Brown 1978, Edelsbrunner 1986).

We present various criteria for a tiling of R^d or, more generally, a PL-manifold in R^d, to be a projection of a (convex) d-surface. These criteria lead to improvements of algorithms determining whether a given tiling can be lifted. We also offer a partial analog of Maxwell correspondence for stresses in 3D frameworks and projections of 3-dimensional surfaces. Our results on generalizations of Maxwell correspondence make use of gain graphs. A gain graph (G, h, H) is a homomorphism h from the free group on the edges of a graph G to some group H; it is called balanced if all closed paths of G lie in the kernel of h. We give a test, formulated in terms of the binary cycle space of G, that can be used to detect if (G, h, H) is balanced for some choices of H, e.g. the abelian case. Some of the discussed results are joint with S. Ryshkov and T. Zaslavski.

For further information contact Janos Pach at pachcims.nyu.edu, or visit our website: http://www.math.nyu.edu/seminars/geometry_seminar.html