Geometry Seminar
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Room 613, Courant Institute
251 Mercer Street, New York

Totally frustrated states: a physics-like generalization of graph coloring

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Abstract

When counting the proper colorations of a graph, whose number is given by the chromatic polynomial, all one needs to know is the number of colors in the color set; the nature of the individual colors is immaterial.x When coloring a gain graph, where the edges are labelled by elements of a group (called the "gain group"), that is no longer so. In order for the concept of a proper coloration to be meaningful the group must have a permutation action on the set of colors, and then the exact way the group acts is a crucial factor in counting proper colorations.

One reason to study gain graph coloring is that it generalizes simple physical models of spin glasses like the Ising model, in which the edges are labelled from the sign group and there are two colors, and the Potts model, in which there are more colors. Due to this connection we call the colors "spins" and the generalized proper coloration a "totally frustrated state".

The number of totally frustrated states is not related to a matroid invariant (as far as I know), except in ordinary graph coloring and a previously known gain-graphic generalization, but it always satisfies a deletion-contraction law and it has a formula in terms of holonomy groups of edge subsets.

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