Geometry Seminar
November 14, 2006, Tuesday, 6:00 p.m.
Room 613, Courant Institute
251 Mercer Street, New York

Neighborly embedded manifolds and continuous hashing

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Abstract

An embedding of an n-dimensional (connected) manifold M into R^d is called *k-neighborly* if for every k points on the embedded manifold there is a hyperplane H in R^d which supports the manifold precisely at these points. Namely, H contains these k points and all other points of the embedded manifold are in the same open half-space determined by H.

Micha A. Perles posed the following problem: What is the smallest dimension $d(k,n)$ of the ambient space in which a k-neighborly n-dimensional manifold exists? A simple dimension count shows that $d(k,n) \geq (k+1)n$. Vassiliev considered a slightly stronger definition of k-neighborly C^2 embeddings, where the k-neighborliness is stable under C^2 perturbations, and showed, by an intricate topological argument, that the analogous function $d'(n,k)$ satisfies $d'(k,n) \geq 2kn - bin(n)$, where $bin(n)$ is the number of ones in the binary expansion of n. A straightforward extension of the moment curve gives an upper bound for $d(k,n)$ which is exponential.

Our main result is: There is a k-neighborly embedding of R^n into R^d for $d = 8(n+1)k^2 + 8k$. The argument is a simple application of a new notion of "continuous hashing". Several continuous analogs of hashing, oriented matroids and polytopes will be discussed. Joint work with Avi Wigderson.

For further information contact {pach,pollack}@cims.nyu.edu, or visit our website: http://www.math.nyu.edu/seminars/geometry_seminar.html