Geometry Seminar
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Room 613, Courant Institute
251 Mercer Street, New York

# On triangles of minimum area and tetrahedra of minimum volume

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**Abstract**

Brass, Rote, and Swanepoel proved that the number of triangles of minimum (nonzero) area determined by $n$ points in the plane is $O(n^2)$, which is asymptotically tight. Here we argue that this bound remains in effect for minimum area triangles in 3-space. We also consider other planar variants of this problem: for sets with no three collinear points, for points in convex position, and acute triangles of minimum area. For the last two variants above, there are only a linear number of such triangles. We then consider the analogous question for tetrahedra in 3-space, where we argue that the number of minimum (nonzero) volume tetrahedra determined by n points is $O(n^3)$, which is asymptotically tight. In the end, we discuss an extension of an algorithm of Edelsbrunner, O'Rourke, and Seidel for reporting all triangles of minimum (nonzero) area, respectively all tetrahedra of minimum (nonzero) volume.

This is joint work with Csaba D. Toth

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