Geometry Seminar
November 9, 2004, Tuesday, 6:00 p.m.
Room 613, Courant Institute
251 Mercer Street, New York

Convex-concave subsets of real projective space

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Abstract

An affine or projective hypersurface is called locally convex if at any point its second order approximation is an ellipsoid. Hopf theorem asserts that a locally convex closed connected hypersurface in R^n is convex.

If the ellipsoid is replaced by a non-degenerate quadric of fixed signature (k, l) we get the class of quasi-convex hypersurfaces. About 15 years ago Arnold suggested a chain of conjectures ultimately leading to creation of "projective topology". The first conjecture claimed that any quasi-convex hypersurface should separate two projective subspaces of complementary dimensions.

Our joint with A. Khovanskii results is the first progress in this direction. First, we prove this conjecture for some particular cases. Second, we introduce a class of projective convex-concave sets – (almost) a subclass of domains bounded by quasi-convex hypersurfaces. These sets retain many properties of convex sets (e.g. duality). We show that the affine analogue of Arnold conjecture is wrong and prove the Arnold conjecture for convex-concave subsets of RP3.

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