Geometry Seminar
October 19, 2004, Tuesday, 6:00 p.m.
Room 613, Courant Institute
251 Mercer Street, New York

Tangents to (various kinds of) objects in R^3

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Abstract

I'll survey a technique that enables to get sharp bounds on the number of lines tangent to various kinds of objects in R^3. Among the kinds we have studied: polytopes (disjoint and in general position at CCCG'02, arbitrary at SCG'04), segments (at CCCG'03) and triangles (at CCCG'04; this work is still in progress). Specifically, we have shown that, under a certain general position assumption, the number of lines tangent to four bounded disjoint convex polyhedra with a total of n edges is $O(n^2)$, and that, under the same assumption, a set of k bounded disjoint convex polyhedra has at most $O(n^2k^2)$ lines, possibly occluded, that are tangent to four of these polyhedra. The same result is valid for arbitrary convex polyhedra (non-disjoint, in degenerate configurations). For segments, we completely describe the structure of the connected components of transversals. We show that $n \geq 3$ arbitrary line segments in R^3 admit 0, 1, …, n or infinitely many line transversals; in the latter case, the transversals form up to n connected components. Finally, we show that four triangles in R^3 may admit at least 88 tangent lines, and at most 216 isolated tangent lines, or an infinity. In the latter case, the tangent lines may form up to 216 connected components, at most 54 of which can be infinite. If no four lines supporting each an edge of a different triangle cannot lie on a common ruled quadric, then the number of tangents is always finite and at most 162; if the four triangles are disjoint, then this number is at most 210; and if both conditions are true, then the number of tangents is at most 156 (the lower bound 88 still applies).

Joint work with (various combinations of) Olivier Devillers,Vida Dujmovic, Hazel Everett, Marc Glisse, Xavier Goaoc, Sylvain Lazard, Hyong-Suk Na, Frank Sottile, and Sue Whitesides.

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