Each problem is worth 10 points (the bonus is 5 points). This homework will be graded out of a total of 30 points. Please state your answers clearly on a separate sheet of paper. Justify your answers.

1

Suppose \( A \) is row equivalent to

\[
\begin{pmatrix}
1 & 2 & 0 & -4 \\
0 & 0 & 1 & -1 \\
0 & 0 & 0 & 0 \\
\end{pmatrix}.
\]

(This is \textit{not} \( A \) itself, just the reduced echelon form of it.)

(i). Find a nonzero vector in the nullspace of \( A \).

(ii). Find a vector in the column space of \( A \).

2

Matrix \( A \) is \( 4 \times 4 \) and invertible. Matrix \( B \) is \( 4 \times 3 \). Suppose \( \text{null}(AB) \) has dimension 2. What is the rank of \( B \)?

Hint: Remember that invertible matrices have \( Ax = 0 \iff x = 0 \). Use this fact to relate \( \text{null}(AB) \) to \( \text{null}(B) \).

3

We have matrices \( A \) and \( B \) with

\[
A = \begin{pmatrix} 3 & 2 & 2 & 1 \\ 1 & 4 & 1 & 1 \\ 2 & 1 & 1 & 1 \\ 1 & 1 & 2 & 1 \end{pmatrix} \quad \text{and} \quad B = \begin{pmatrix} 3 & 2 & 2 & 1 \\ 1 & 4 & 1 & 1 \\ 2 & 1 & 1 & 1 \\ 0 & 3 & -1 & 0 \end{pmatrix}.
\]

We know that \( \det(A) = -7 \). Find \( \det(B) \).

Hint: \( A \) and \( B \) are row equivalent. Make sure to show your work.

\textbf{Bonus (5pts)}

Find a matrix \( A \) so that \( \text{col}(A) = \text{null}(A) \).