Pricing Energy Market Quanto Options

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Volume risk is a big concern for energy companies, especially since prices and volume are highly correlated.

Weather derivatives have served as tools to hedge volume.

The recent decrease in traded volume (from 930,000 in 2007 to under 500,000 in 2009) of this is attributed to co-called "Energy quantos".

A bit different from currency quantos as the price is not denominated in something else - it more has the structure of a "double option", e.g.,

\[ \max(S_1 - K_1, 0) \times \max(S_2 - K_2, 0) \]
Change in payoff compared to pure gas options

Pure gas options vs. quanto option
Energy Quantos provide tailormade risk management products

OTC trading is not that much of a compromise, since many weather derivatives are illiquid

Counterparts can be insurance companies, reinsurance companies, other energy companies
Scarce literature on Energy Quantos

- Paper by Caporin, Pres & Torró (Energy Economics 2012)
  - A bivariate time series model to capture the joint dynamics of energy prices and temperature.
  - Parameter-intensive econometric model
  - MC simulation to calculate prices.
  - Hedging goes unanswered

- Several papers showing the link between energy prices and temperature

- Several papers on seasonality, modelling of energy prices etc.
Contribution

- Energy and weather futures are traded with delivery periods.
- Typically, energy quanto options have a payoff which can be represented as an "Asian" structure on the energy spot price and the temperature index.
- Hence, any "Asian payoff" on the spot and temperature for a quanto option can be viewed as a "European payoff" on the corresponding futures contracts.
- We convert the pricing problem by using futures contracts as underlying assets, rather than energy spot prices and temperature.
- Derive analytical solutions to the option pricing problem (in a log normal framework) as well as hedging strategies in terms of the traded underlying futures contracts.
- Estimate our model using NYMEX data.
Example of contract

Generally, Energy Quantos are tailormade. For instance:

<table>
<thead>
<tr>
<th></th>
<th>Nov</th>
<th>Dec</th>
<th>Jan</th>
<th>Feb</th>
<th>Mar</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a) High Strike</td>
<td>$K_{11}^H$</td>
<td>$K_{12}^H$</td>
<td>$K_1^H$</td>
<td>$K_2^H$</td>
<td>$K_3^H$</td>
</tr>
<tr>
<td>(b) Low Strike</td>
<td>$K_{11}^L$</td>
<td>$K_{12}^L$</td>
<td>$K_1^L$</td>
<td>$K_2^L$</td>
<td>$K_3^L$</td>
</tr>
<tr>
<td>(a) High Strike</td>
<td>$K_{11}^E$</td>
<td>$K_{12}^E$</td>
<td>$K_1^E$</td>
<td>$K_2^E$</td>
<td>$K_3^E$</td>
</tr>
<tr>
<td>(b) Low Strike</td>
<td>$K_{11}^E$</td>
<td>$K_{12}^E$</td>
<td>$K_1^E$</td>
<td>$K_2^E$</td>
<td>$K_3^E$</td>
</tr>
<tr>
<td>Volume (mmBtu)</td>
<td>200</td>
<td>300</td>
<td>500</td>
<td>400</td>
<td>250</td>
</tr>
</tbody>
</table>

Table: As an example the payoff for November will be: (a) In cold periods $\max(H - K_H, 0) \times \max(E - K_E, 0) \times \text{Volume}$. (b) In warm periods $\max(K_H - H, 0) \times \max(K_E - E, 0) \times \text{Volume}$. We see that the option pays out if both the underlying temperature and price variables exceed (dip below) the high strikes (low strikes).
From Asian to European

- A futures contract $F^E(\tau_2, \tau_2)$ promises to deliver gas at a constant rate over the delivery period $[\tau_1, \tau_2]$.
- Defining a quanto option on the average $\sum_{u=\tau_1}^{\tau_2} S_u$ therefore corresponds to writing the option on the futures contract.
- The same argument holds for the temperature index, where we specifically use futures on Heating Degree Days (how much the temperature is below 18 degrees C).
- Specifically, we study a “double option”-structure:

$$\hat{p} = \max \left( F^E_{\tau_2}(\tau_1, \tau_2) - K_E, 0 \right) \times \max \left( F^I_{\tau_2}(\tau_1, \tau_2) - K_I, 0 \right)$$
General setup

The two futures prices are log-normal under the pricing measure $\mathbb{Q}$

$$F^E_T(\tau_1, \tau_2) = F^E_t(\tau_1, \tau_2) \exp(\mu_E + X)$$

$$F^I_T(\tau_1, \tau_2) = F^I_t(\tau_1, \tau_2) \exp(\mu_I + Y)$$

where

- $(X, Y)$ is a bivariate normally distributed random variable with mean zero, with covariance structure depending on $t, T$ and $\tau_2$.
- $\sigma^2_X = \text{Var}(X), \sigma^2_Y = \text{Var}(Y)$ and $\rho_{X,Y} = \text{corr}(X, Y)$.
- The futures price naturally is a martingale under the pricing measure $\mathbb{Q}$, so $\mu_E = -\sigma^2_X/2$ and $\mu_I = -\sigma^2_Y/2$.
- This structure encompasses many models, e.g., geometric Brownian motion models and multi-factor spot models.
Option price

The time $t$ market price of an European energy quanto option with exercise at time $\tau_2$ and payoff described on the previous slides

$$C_t = e^{-r(\tau_2-t)} \left( F_t^E e^{\rho_X,\gamma\sigma_X\sigma_Y} M(y_{***}^{*}, y_{**}^{*}; \rho_X,\gamma) - F_t^E K_I M (y_{**}^{*}, y_{**}^{*}; \rho_X,\gamma) - F_t^I K E M (y_{*}^{*}, y_{*}^{*}; \rho_X,\gamma) + K_I K E M (y_{1}, y_{2}; \rho_X,\gamma) \right)$$

$$y_1 = \frac{\log \left( \frac{F_t^E}{K_E} \right) - \frac{1}{2} \sigma_X^2}{\sigma_X}, \quad y_2 = \frac{\log \left( \frac{F_t^I}{K_I} \right) - \frac{1}{2} \sigma_Y^2}{\sigma_Y},$$

$$y_{1}^{*} = y_1 + \rho_X,\gamma \sigma_Y, \quad y_{1}^{**} = y_1 + \sigma_X, \quad y_{1}^{***} = y_1 + \rho_X,\gamma \sigma_Y + \sigma_X,$$

$$y_{2}^{*} = y_2 + \sigma_Y, \quad y_{2}^{**} = y_2 + \rho_X,\gamma \sigma_X, \quad y_{2}^{***} = y_2 + \rho_X,\gamma \sigma_X + \sigma_Y.$$
Temperature data

Temperature contracts

- Futures contracts on Heating Degree Days are traded for several cities for the months October to April.
- The contract value is 20$ for each HDD in the month. The underlying is one month of accumulated HDD’s for a specific location.
- The futures price is denoted by $F_t^I(\tau_1, \tau_2)$ and settled on the index $\sum_{u=\tau_1}^{\tau_2} HDD_u$.
- Estimation is done using the first seven contracts where the index period hasn’t startet yet.
- The chosen locations are New York and Chicago, since these are located in an area with fairly large gas consumption.
Gas data

Gas contracts

- Futures contracts for delivery of gas is traded on NYMEX for each month ten years out.
- The underlying is delivery of gas throughout a month and the price is per unit.
- Estimation is done using the first 12 contracts for delivery at least one month later.
Chosen model: Sørensen (2002)

In the Sørensen (2002) model, the Schwartz-Smith (2000) two-factor model is extended to include seasonality:

$$\log F_t(\tau) = \Lambda(\tau) + A(\tau - t) + X_t + Z_t e^{-\kappa(\tau - t)},$$

where $A(\tau) = \alpha \tau - \frac{\lambda Z - \rho \sigma \nu}{\kappa} (1 - e^{-\kappa \tau}) + \frac{\nu^2}{4\kappa} (1 - e^{-2\kappa \tau}).$

The two factors and the seasonality function:

$$dX_t = \left( \alpha - \frac{1}{2} \sigma^2 \right) dt + \sigma dW_t,$$

$$dZ_t = - (\lambda Z + \kappa Z_t) dt + \nu dB_t,$$

$$\Lambda(t) = \sum_{k=1}^{K} (\gamma_k \cos(2\pi kt) + \gamma_k^* \sin(2\pi kt)).$$
Extending to two assets

- Two futures dynamics of this are connected by allowing the Brownian motions to be correlated across assets:

\[
\rho_E = \text{corr}(W^E_1, B^E_1), \quad \rho_I = \text{corr}(W^I_1, B^I_1),
\]

\[
\rho_W = \text{corr}(W^E_1, W^I_1), \quad \rho_B = \text{corr}(B^E_1, B^I_1).
\]

- Joint dynamics of the futures price processes \(F^E_t(\tau_1, \tau_2)\) and \(F^I_t(\tau_1, \tau_2)\) under \(\mathbb{Q}\) is given by (with \(\eta_i(t) = \nu_i e^{-\kappa_i(\tau_1-t)}\))

\[
\frac{dF^i_t(\tau_1, \tau_2)}{F^i_t(\tau_1, \tau_2)} = \sigma_i dW^i_t + \eta_i(t) dB^i_t,
\]

for \(i = E, I\) and can thus be represented as

\[
F^E_T(\tau_1, \tau_2) = F^E_t(\tau_1, \tau_2) \exp (-\mu_E + X).
\]
### Estimates

<table>
<thead>
<tr>
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<th>New York</th>
<th>Gas</th>
<th>Chicago</th>
<th>Gas</th>
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<tbody>
<tr>
<td>$\kappa$</td>
<td>16.5654 (1.1023)</td>
<td>0.6116 (0.0320)</td>
<td>18.8812 (1.3977)</td>
<td>0.6034 (0.0317)</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>0.0494 (0.0059)</td>
<td>0.2342 (0.0200)</td>
<td>0.0379 (0.0051)</td>
<td>0.2402 (0.0209)</td>
</tr>
<tr>
<td>$\nu$</td>
<td>3.6517 (0.6197)</td>
<td>0.6531 (0.0332)</td>
<td>4.3980 (0.8908)</td>
<td>0.6647 (0.0335)</td>
</tr>
<tr>
<td>$\rho$</td>
<td>$-0.6066$ (0.0801)</td>
<td>$-0.6803$ (0.0656)</td>
<td>$-0.5509$ (0.0948)</td>
<td>$-0.7038$ (0.0611)</td>
</tr>
<tr>
<td>$\sigma_\epsilon$</td>
<td>0.0655 (0.0006)</td>
<td>0.0199 (0.0001)</td>
<td>0.0554 (0.0005)</td>
<td>0.0199 (0.0001)</td>
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<tr>
<td>$\gamma_1$</td>
<td>0.9044 (0.0023)</td>
<td>0.0500 (0.0003)</td>
<td>0.8705 (0.0019)</td>
<td>0.0499 (0.0003)</td>
</tr>
<tr>
<td>$\gamma_1^*$</td>
<td>0.8104 (0.0018)</td>
<td>0.0406 (0.0003)</td>
<td>0.6391 (0.0015)</td>
<td>0.0406 (0.0003)</td>
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<tr>
<td>$\gamma_2$</td>
<td>N/A</td>
<td>0.0128 (0.0003)</td>
<td>N/A</td>
<td>0.0128 (0.0003)</td>
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<tr>
<td>$\gamma_2^*$</td>
<td>N/A</td>
<td>0.0270 (0.0003)</td>
<td>N/A</td>
<td>0.0270 (0.0003)</td>
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<tr>
<td>$\rho^W$</td>
<td>$-0.2843$ (0.0904)</td>
<td></td>
<td>$-0.2707$ (0.0909)</td>
<td></td>
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<tr>
<td>$\rho^B$</td>
<td>0.1817 (0.0678)</td>
<td>0.1982 (0.0643)</td>
<td></td>
<td></td>
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<tr>
<td>$\ell$</td>
<td>36198</td>
<td>37023</td>
<td></td>
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</table>
Fit of the model (New York and gas)

Figure: Model prices (blue) and observed prices (green) for the joint estimation of Natural Gas Futures and New York HDDs
RMSE for New York HDD futures
RMSE for gas

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**Benth, Lange & Myklebust**

**Quanto Options**
### Example of derived option prices

<table>
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<tr>
<th>$K_1/K_G$</th>
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<td>1100</td>
<td>596</td>
<td>451</td>
<td>337</td>
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<td></td>
<td>470</td>
<td>355</td>
<td>270</td>
<td>206</td>
<td>158</td>
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<tr>
<td>1150</td>
<td>443</td>
<td>338</td>
<td>254</td>
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<td>110</td>
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<td>401</td>
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<td>222</td>
<td>168</td>
<td>127</td>
<td>97</td>
<td>74</td>
</tr>
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</table>

**Table**: Option prices for Chicago under the model (top) and under the assumption of no correlation (bottom). $r = 0.02$, $\tau_1=1$-Dec-2011, $\tau_2=31$-Dec-2011, $t=31$-Dec-2010
Change in payoff compared to pure gas options

Pure gas options vs. quanto option
Quanto deals have decreased the volume of weather derivatives trading
Quanto deals are good tools for tailoring the risk management of volume risk
Derived closed form pricing and hedging formulas in log normal model
A two asset two factor model was applied to NYMEX data