Statement of Teaching and Learning

In my three years of teaching, I have instructed engineering students from The Cooper Union (in Ordinary Differential Equations, Boundary Value Problems, Probability and Statistics) and a variety of undergrads at New York University (in Quantitative Reasoning). For each of these classes, I see myself as a coach: an expert, a motivator, and a diagnostician. I try to create an effective learning environment through decisions informed by research in cognition and experience in instruction.

When designing a class, I consider four types of questions:

- Knowledge-centered, i.e. What do I want my students to be able to do?
- Learner-centered, i.e. How do I help students build on and refine their existing mental models?
- Assessment-centered, i.e. How can I frequently reveal the progress students have made and the work yet to be done in achieving the course goals?
- Community-centered, i.e. How do I capitalize on the presence of a community of learners and the community at large?

Knowledge-Centered Aspects

At the beginning of a course, I consider what I actually want my students to be able to do. I provide written objectives that are specific enough to guide student thinking but general enough to promote transfer of learning to new contexts. Making these goals explicit led me to realize that my first attempts at teaching were inefficient at causing the transformation I desired in my students.

As an example, consider the attached objectives for my second time teaching Boundary Value Problems (BVP). At the end of the first year, I noticed most students were able to solve the partial differential equations but were unable to relate them to the physical situations they describe. The fault, here, lied with me: I implicitly assumed my students would figure out this important skill as a byproduct of solving many homework problems. When I taught the course a year later, I decided the primary goal of the class was for students to pose and interpret these equations. Making this and other objectives explicit led me to overhaul the course’s organization, the in-class activities, and the kinds of homework and exam questions I asked. Notice that in the attached “Homework from Day 16,” the problems are directed squarely at all three objectives.

Learner-Centered Aspects

After I have fleshed out a course’s objectives, I can tend to the development of my students’ mental models and skills. The fundamental principle here is the constructive nature of knowing: that we actively build understanding on top of our existing mental structures. In contrast to a belief that information is simply transmitted from teacher to student, the constructivist view requires a keen awareness of whether students have the knowledge with which to understand the presented content. Papers by John Bransford and Daniel Schwartz were influential in guiding me to ask the question “How can I prepare students to understand a mathematical abstraction?”
One such way is through course organization. Consider the attached syllabus for Boundary Value Problems. It does not begin with the mathematically simplest problems; instead, it begins with the physical experience students bring to the class. After developing the interesting qualitative questions about diffusions and waves, we began to see how many we can answer only with basic physical assumptions. Then we saw how many we can answer by inspection of a differential equation. Only after that did we deal with the extremely technical analysis of these equations.

Another way I prepare students for abstraction is through homework exercises that create the background knowledge needed in class. In the attached “Homework from Day 16,” problem 2 concerns the technique of separation of variables in solving a diffusion equation. It guides students to contrast the properties of an example from class with two variants for which the same technique fails. This problem provides them with the differentiated knowledge that prepares them to be told the mathematical generalization in class.

In addition to these high level thought processes, I try not to overlook the necessity for developing automaticity in each discipline’s most basic skills. For example, in my BVP course, I found individual whiteboards to be an excellent medium for such tasks. Each day of class, we would begin with a 2-3 minute whiteboard warm-up question, such as “Write down a diffusion equation with homogeneous Dirichlet boundary conditions.” At first, students would be flipping through their notes for the definition of “homogeneous” and “Dirichlet,” but as the semester progressed their recall speed greatly improved. Once my students had automaticity with these important definitions, I was comfortable using the terms in class without any extra pause or reminder.

The final learner centered aspect I will discuss is motivation. As an instructor, I have the capacity to transform my students only if they care. A significant part of my job, then, is to convince them that the course’s content is worth knowing. The challenge was greatest for my course in Quantitative Reasoning, a graduation requirement at NYU. In order to convince the students that the skills of estimation could actually help them, I had to show them that their own wild guesses can be completely wrong. On the second day of class, everyone unanimously agreed that we have many more brain cells than bytes on a typical hard disk and were duly surprised to hear the opposite is true. A five-minute activity like this can greatly alter how receptive the students are to the technicalities of estimation.

Assessment-Centered Aspects

Since my courses are about meeting prescribed objectives, assessment naturally plays the role of informing my students and me about their progress. Ideally, I would like a day-to-day formative assessment of my students’ ability so that we can notice misunderstandings before it is too late. For example, in my BVP class, I can give a whiteboard problem about how they would plan to solve a given differential equations. After a minute or so of writing, they show me their boards, and I provide immediate, individualized feedback on their methods.

For assessments that affect grades, I believe that students should have multiple opportunities to demonstrate competence with the course objectives. If they fail at a task, a second or third chance will make them more likely to learn from their mistakes and metacognitively analyze their understanding of the course content. In my probability class, the difference between homework revisions was striking: I watched explanations evolve from being verbose and confusing to being clear and direct.

Even with exams, I often allow multiple revisions. In my BVP class, the final was an oral interview in line with the course objectives: I asked them to provide specific examples that illustrate the important qualitative
properties of partial differential equations. Many students provided physically ridiculous responses by over-relying on some mathematics they didn’t understand. In these cases, I would point out how their response contradicts physical observations, and point them to relevant course material they should further analyze. After that, I encouraged them to schedule another final. One student said of the revision process:

I went to the exam expecting to do extremely well, ... [but] I was shocked when I failed the oral final not only once, but twice. During revision, a series of questions was posed to guide me through the process of crystallizing physical phenomena into well defined problems and identifying the appropriate mathematical tools to arrive at meaningful solutions. The course completely changed the way I approach various problems I encounter. Instead of just memorizing how a particular theorem is derived or how an equation is solved, I always strive to understand why this particular technique is used, when its application becomes inappropriate, and what the solution means.

On the third try at the final, this student’s explanations and examples were outstanding.

Community-Centered Aspects

In order to capitalize on the community of learners in the classroom, I provide in-class activities that allow individual and small group thinking as preparation for a whole-class discussion. For example, in my *Quantitative Reasoning* class, I handed out a faulty calculation of how many bacteria stacked end-to-end it would take to span from Earth to the moon. The purported answer was $3.8 \cdot 10^2$. In this Think/Pair/Share activity, students had 1 minute to individually come up with more than one way to notice the given answer was incorrect. Then, they had 2 minutes to compare and explain their thoughts to their neighbors. After that, we had a short whole-class discussion on the topic. As there were several ways to analyze the calculation, peers can provide new viewpoints to each other. Students will be much more likely to remember these multiple ways than if I had simply written them on the board.

What’s to Come

The best course decisions create a synergy between several, if not all, of these four major aspects of instruction. For example, the course objectives guide what I emphasize, what misunderstandings I look for, and what is worth assessing. Similarly, personal white-board activities in class highlight the important content, allow students to construct and reveal their understanding, and provide immediate peer and instructor feedback.

I have adopted these four major considerations only after much reading and thought in human instruction. Not surprisingly, answers to the above issues motivate more questions: What kinds of experiences best induce abstraction in students? And how can homework be usefully and fairly tailored to individuals? I am also interested in broader changes: How and when should I create a more student-directed class? And how do I ensure the course objectives are met in such a class? I hope to find answers to some of these questions by reading the research literature and trying out ideas when local conditions permit. In the meantime, I will continue asking myself the knowledge-, learner-, assessment-, and community-centered questions for every course I teach.
Boundary Value Problems

Objectives

By the end of the course you will be able to:

1. Pose physical situations involving diffusions and wave propagation as boundary value problems and interpret such problems physically.

2. Demonstrate the qualitative behaviors of the diffusion and wave equations through explicit examples.

3. Isolate and explain the main idea and conditions of applicability of several PDE solution techniques.

Syllabus

The course will be broken into the following segments:

I.) Posing physical situations as BVPs
   
   (i) Posing the major qualitative questions about diffusions and wave propagation.
   
   (ii) Using first principles (conservation of mass, Fick’s Law, Newton’s Law, etc.) to assess the reasonability of certain qualitative behaviors.

   (iii) Using diffusion and wave PDEs to assess the reasonability of certain qualitative behaviors.

   (iv) Posing and interpreting boundary value problems.

II.) Solution techniques
   
   (i) Symmetry and change of coordinates.

   (ii) Travelling wave solutions and similarity solutions.

   (iii) Fourier series and Fourier transform.

   (iv) Eigenfunction expansions and separation of variables.

   (v) Dirac Delta, Green’s functions, and method of images.
1. Consider a guitar string of tension $T$, length $L$, linear density $\rho$, and ends held fixed. Suppose the string is initially held fixed at a displacement $u(t = 0, x) = \sin \frac{\pi x}{L}$ and released.

(a) Write down the BVP satisfied by the displacement $u(t, x)$.
(b) Draw a sequence of profiles which you expect to correspond to the behavior of the string. What is a natural question to ask about this system concerning the sound produced?
(c) Solve the BVP from (a) by separation of variables.
(d) Use the exact solution from (c) to answer your question from (b).

2. In class, we found that we could solve

$$\partial_t u - D\partial_{xx} u = 0$$

$$u|_{t=0} = \sin \frac{n\pi x}{L}$$

$$u|_{x=0} = 0$$

$$u|_{x=L} = 0$$

by separation of variables, where $n$ is an integer.

(a) Would the same technique work if the initial data were $u|_{t=0} = \sin \frac{\pi x}{2L}$?
(b) Would the same technique work if the initial data were $u|_{t=0} = x(L - x)$?
(c) Use (a) and (b) to determine why separation of variables works in the case $u|_{t=0} = \sin \frac{n\pi x}{L}$.