Thoughts on Individuation

College level mathematics courses are conventionally taught with a fixed sequence of homework problems which all students must complete by given dates. Usually, homework acts as summative assessment and is unresponsive to student development. If, for example, students performed poorly on a given homework problem, they are expected to identify and correct this deficiency on their own. As they were unable to master the content before the homework due date, they likely need guidance in developing and executing new learning strategies. Students can be motivated to work in their Zone of Proximal Development (ZPD) by having them control and document their progress toward specific learning objectives. In this document, we will discuss possible ways to give students this power and to get them to use it.

Given the possible range of students’ mathematics knowledge, tasks that can be individually tailored have the promise to engage all students in their ZPD. With variable-difficulty tasks and instructor guidance, students could control their progress toward toward the course objectives. In order to design such tasks, we will distinguish between explicit and implicit variability. An explicitly variable task might include a sequence of related problems with progressively less scaffolding. Students could begin at whatever difficulty they decide, knowing the course objective is to solve the unscaffolded problem. A task with implicitly variable difficulty is a fixed statement which each student will execute differently. In tasks that involve making problems precise or planning solution strategies, students will naturally view the problem through their well-learned schemata, which may be insufficient for the task at hand.

To illustrate an explicitly variable task, consider one day’s learning objectives from my Quantitative Reasoning class at NYU:

- To determine the present value (PV) of a series of future payments.
- To identify and evaluate geometric sums in such calculations.

A sequence of exercises after class might like:

1.) Find the PV of a prize that pays $10,000 in 2 year from now.
2.) Find the PV of a prize that pays $10,000 a year for the next 3 years.
3.) Find the PV of a prize that pays $10,000 a year for the next 50 years.
4.) Find the PV of a prize that pays $10,000 a year between 2010-2059.

Each of these problems introduces an additional layer of complexity, building up to the day’s objectives.

To illustrate an implicitly variable task, consider two objectives from my course in Boundary Value Problems:

- To pose physical situations involving diffusion as a boundary value problem and to interpret such problems physically.
- To demonstrate the qualitative behaviors of the diffusion equation through explicit examples.

A goal-free homework problem related to these objectives could be:

1.) Consider a closed tube of liquid with dye concentrated at the center. What qualitative claims can you make about the system? Back up as many as you can with an explicit solution to the relevant BVP.
The mathematical difficulty of this problem will vary between students because of their differing schemata for diffusion and mathematical techniques. The solutions will readily identify to the instructor where each student should direct his or her effort.

Another example of an implicitly variable task is a multi-week project in mathematical neuroscience for advanced undergraduates. Students could be presented with the following imprecise problem statement:

- In the Hodgkin-Huxley model of the squid giant axon, an action potential arises due to the interaction between potassium, sodium, and other ion channels. If we injected a drug that blocked 75% of the sodium channels, how would that affect the electrical behavior of the neuron?

Especially to a newcomer to the field, this problem is daunting. The ability to break hard problems into simpler ones, to seek out resources, and to identify barriers to progress are important skills that are not usually taught in mathematics. Hence, students should exhibit a wide range of ability in these skills, and individual learning experiences should be much more efficient than conventional ones.

Unfortunately, allowing students to control their learning does not mean they will actually choose to do so. As intellectual development requires the realization that one lacks knowledge or skills, even good students may perform below their ability. Heavily relying on extrinsic motivators, such as grades, may decrease intrinsic motivation and create an intellectually risk-averse climate. Additionally, difficulty variability in assignments may appear to be an arbitrary use of instructor power.

An alternative way to motivate students is to have them document their progress toward the course objectives. For example, in the QR homework above, students could be asked to return to class with either a solution to the problems or the barrier that is preventing them from progressing. Similarly, in the neuroscience project, students could create a plan in class for how to attack the project. At the next class, they can return with work toward that plan and the barrier against more progress.

Clearly and continually documenting progress is motivating for several reasons. Firstly, writing down barriers to progress makes tasks easier by clarify what is difficult about them. Secondly, students will be unlikely to knowingly and repeatedly perform below their perceived skill level because the documentation will produce a dissonance with their sense of ability. Finally, clearly documented progress of difficult learning will create a stronger sense of accomplishment when students reflect over the problem.

While documentation alone should help students, it will be most effective when it guides the instructor’s feedback. For example, in the goal-free problem above, students may return to class with a list of qualitative properties which they do not know how to quantify. At that point, the instructor and student can create a plan which the student can execute and return with progress or a the barrier to it.

Individuated instruction does not mean that different students are held to different standards. Its intent is for all students to meet the same course objectives, but beginning at the individual’s level of understanding. Assignments like those mentioned above introduce logistical challenges beyond those of conventional homework sets. In particular, if every student is working at different tasks and rates, what communal instructions or activities will be most useful? Nonetheless, making instruction more individual could result in much more effective, long-lasting learning.