

C.V. of Joseph Coffey

November 29, 2004

Research Interests

Symplectic Topology

I am symplectic geometer, currently at an NSF postdoctoral fellowship at Courant. My thesis, and some of my current work, looks at symplectic geometry through the lens of algebraic topology. More recently I have worked on perspectives from geometric measure theory. Currently I am trying to understand connections between convex surfaces in contact 3-manifolds and holomorphic curves in their symplectization.

Education

SUNY Stony Brook - Ph.D. in Mathematics, Aug 2002.

Advisors: Professors Dennis Sullivan and Dusa McDuff.

- Thesis title: Symplectomorphism groups and isotropic skeletons and Maps with symplectic graphs
- Math GPA: 4.0/4.0
- Orals:
 - Major: Partial differential relations, Mikhael Gromov
 - Minor: Psuedo-holomorphic curves in symplectic manifolds, Mikhael Gromov (Passed with distinction)

University of California, Berkeley - BA in Mathematics, May 1996 (With Honors)

- Graduate Course-work in : Algebraic Geometry (3 courses), Commutative Algebra, Von Neuman Algebras, Lie Groups and others.

University of California, Berkeley - BA in Mathematics, May 1996 (With Honors)

Awards and Fellowships

- Stony Brook University Graduate Fellowship 1996-2001
- NSF Postdoctoral Fellowship 2002 – 2005

Teaching/Work Experience

- Lectured the following courses at New York University:
 - Pre-Calculus, Spring 2003
 - Business Calculus, Fall 2003

- Calculus I, Spring 2004
- '97-2002 Teaching Assistant at SUNY
 - TA for one variable calculus courses from “Business Calculus” through Honors Calculus: 122,123,131,142
 - TA for multi- variable calculus course 205
 - TA for Differential Equations course 303
 - Lecturer/Course Coordinator for Trigonometry course 130
 - Substitute lecturer for Calculus: 131,122 and Abstract Algebra 318
 - Statistical Review of Math Placement Exam- Commissioned by department to aid in review of Math Placement Exam, the exam which decides which students take which courses. Did statistical analysis of exam questions.
 - Over 400 hours tutoring Students of all levels (pre-algebra through upper division courses) in Math Learning Center
- '95 Berkeley High School, Berkeley, CA
 - Teaching Assistant for 2 physics classes, via Education Class at UC Berkeley

Expository Lectures

- Sobolev Spaces in Symplectic Topology I and II, Symplectic Seminar '96
- Eliashberg's h-principle for Folded Maps, Symplectic Seminar '97
- J-holomorphic Curves in Symplectic Topology, Graduate Seminar '97
- Donaldson's Construction of codimension 2 symplectic submanifolds, Symplectic Seminar '99
- Hahn-Banach Theorem in Geometry and Topology (after Dennis Sullivan), Graduate Seminar 2001
- Convex surfaces in contact topology (after Giroux, Honda), Spring 2004
- Planar open book decompositions in contact topology (after Etnyre and Eliashberg) Spring 2004

Lectures on Research

- Lefschetz pencils and symplectomorphism groups, Graduate Seminar 2000
- Which Lefschetz pencils determine a symplectic Structure? Topology-Geometry Seminar 2001
- Symplectomorphism groups of 4 Manifolds, Topology-Geometry Seminar (scheduled)2002
- Symplectomorphism groups of 4 Manifolds, Symplectic Seminar (scheduled) 2002
- Maps with symplectic graphs, Instituto Technico, Lisbon Portugal, Fall 2002
- Maps with symplectic graphs, Courant-Stony Brook, Joint symplectic seminar Spring 2003
- Measured J-holomorphic foliations, Courant symplectic seminar, Spring 2004
- Failure of the parametric h-principle for maps with prescribed Jacobian, AMS regional meeting, Los Angeles, CA, Feb 2004
- Failure of the parametric h-principle for maps with prescribed Jacobian, Columbia - Gauge theory seminar, Oct 2004

Papers and preprints

Copies of all of these are available at my web site: math.nyu.edu/~coffey.

Symplectomorphism groups and isotropic skeletons

(Revision of exposition submitted to Geometry and Topology)

The symplectomorphism group of a 2-dimensional surface is homotopy equivalent to the orbit of a filling system of curves. We give a generalization of this statement to dimension 4. The filling system of curves is replaced by a decomposition of the symplectic 4-manifold (M, ω) into a disjoint union of an isotropic 2-complex L and a disc bundle over a symplectic surface Σ which is Poincare dual to a multiple of the form ω . We show that then one can recover the homotopy type of the symplectomorphism group of M from the orbit of the pair (L, Σ) . This allows us to compute the homotopy type of certain spaces of Lagrangian submanifolds, for example the space of Lagrangian $\mathbb{R} \mathbb{P}^2 \subset \mathbb{C} \mathbb{P}^2$ isotopic to the standard one. This paper was a portion of the authors PhD thesis under Dusa McDuff and Dennis Sullivan

Failure of the parametric h-principle for maps with prescribed Jacobian

Let M and N be closed n -dimensional manifolds, and equip N with a volume form σ . Let μ be an exact n -form on M . Arnold then asked the question: When can one find a map $f: M \rightarrow N$ such that $f^*\sigma = \mu$. In 1973 Eliashberg and Gromov showed that this problem is, in a deep sense, trivial: It satisfies an h-principle, and whenever one can find a bundle map $f_{bdl}: TM \rightarrow TN$ which is degree 0 on the base and such that $f_{bdl}^*(\sigma) = \mu$ one can homotop this map to a solution f . That is if the naive topological conditions are satisfied one can find a solution. There is no further interesting geometry in the problem.

We show the corresponding parametric h-principle fails- if one considers *families* of maps inducing μ from σ , one can find interesting topology in the space \mathcal{G}_μ of solutions which is not predicted by an h-principle. Moreover the homotopy type of such maps is “quantized”: for certain families of forms homotopy type remains constant, jumping only at discrete values.

Spaces of maps with symplectic graph

(Submitted to Commentari Math Helv.)

We consider the homotopy type of the space $\mathcal{M}^\sigma(\Sigma, \Gamma)$ of maps between symplectic surfaces (Σ, σ_Σ) and (Γ, σ_Γ) whose graphs form symplectic submanifolds of the product. We give a purely topological model for this space in terms of maps with constrained numbers of pre-images. We use this to show that the dependence of the homotopy type on the forms σ_Σ and σ_Γ is quantized- it changes only when the parameters pass certain discrete levels. When the domain Σ is a sphere or torus, and further $\sigma_\Sigma(\Sigma) \leq \sigma_\Gamma(\Gamma)$, we compute the full homotopy type of the low degree components. We also give an example, showing that the homotopy type of the space of symplectic sections of a symplectic fibration F must sometimes change as we deform F . Much of this work generalizes to n -dimensional manifolds equipped with volume forms. Some of this material (about half) was contained in the authors PhD thesis under Dusa McDuff and Dennis Sullivan.

Measured J-holomorphic foliations

Let Σ denote an oriented surface. The Narisham-Sheshadri theorem, suitably translated, says that on $S^2 \times \Sigma$ for any integrable complex structure j which makes the sections $0 \times \Sigma$ and $\infty \times \Sigma$ holomorphic there is a 1 complex dimensional foliation such that the leaves are j -invariant and the foliation has an invariant metric.

We show that this does not hold in the almost complex category. We produce a j on $S^2 \times \Sigma$ where there can be no such foliation, by proving a *uniqueness* theorem for foliations with smooth invariant *measures*. Then we prove an *existence* theorem for a less regular, but analogous object via an extension of Sullivan's Hahn Banach alternative.

We show that when an almost complex structure admits sufficient symmetry the objects we produce are foliations with smooth invariant measures and thus unique. However in general this uniqueness depends subtly on the regularity of the resulting "foliation". Finally we show that constructions relating J -holomorphic curves and almost complex structures on rational surfaces admit generalizations within in this context.

Letters of Reference

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