Discrete Math
Midterm
No Books or Calculators
Explain all answers

1. (10 pts) \( \sum_{k=0}^{8} \frac{8!}{k!(8-k)!}(-1)^k2^{8-k} = 1^8 = 1 \)

2. (10 pts) Describe an algorithm that has worst-case complexity of \( \theta(n\log n) \).
   Prove your statements.
   Merge sort is an example we did in class.

3. (10 pts) In a class of 12 students, half of whom are juniors, how many different groups of three students with at least one junior in them?
   There are \( \frac{6!}{3!3!} = 20 \) combinations of groups of 3 with no junior in them, and \( \frac{12!}{3!9!} = 220 \) combinations in all, and the difference between the two, 200 is the number we want.

4. (10 pts) Give the best lower bound for the number of modulus operations required to find the greatest common divisor of two 9 digit numbers. You may leave your answer in functional form.
   \( \log_2 \frac{2 \times 10^9}{3} \)

5. (10 pts) Are there two integer numbers \( x \) and \( y \) (not necessarily positive), such that \( 42x + 35y = 1 \)? If so, find all such pairs.
   No! The left-hand side is divisible by 7, the right hand is no.