1. Suppose the positive integer \( n \) is odd. First Al writes the numbers 1, 2, \ldots, 2n on the blackboard. Then he picks any two numbers \( a, b \), erases them, and writes, instead, \(|a - b|\). Prove that an odd number will remain in the end.

2. A circle is divided into 6 sectors. Then the numbers 1, 0, 1, 0, 0, 0 are written into the sectors (counter-clockwise, say). You may increase two neighboring numbers by 1. Is it possible to equalize all numbers by a sequence of such steps?

3. In the Parliament of Sikinia, each member has at most three enemies. Prove that the house can be separated into two houses, so that each member has at most one enemy in his own house.

4. Suppose not all four integers \( a, b, c, d \) are equal. Start with \((a, b, c, d)\) and repeatedly replace \((a, b, c, d)\) by \((a - b, b - c, c - d, d - a)\). Then at least one number of the quadruple will eventually become arbitrarily large.

5. Each of the numbers \( a_1, a_2, \ldots, a_n \) is 1 or \(-1\), and we have
   \[ S = a_1a_2a_3a_4 + a_2a_3a_4a_5 + \ldots + a_na_1a_2a_3 = 0. \]
   Prove that \( 4 \mid n \).

6. \( 2n \) ambassadors are invited to a banquet. Every ambassador has at most \( n - 1 \) enemies. Prove that the ambassadors can be seated around a round table, so that nobody sits next to an enemy.

7. Start with the set \( \{3, 4, 12\} \). In each step you may choose two of the numbers \( a, b \) and replace them by \(0.6a - 0.8b\) and \(0.8a + 0.6b\). Can you reach the goal \( (a, b) \) in finitely many steps:
   (a) \( \{4, 6, 12\} \),
   (b) \( \{x, y, z\} \) with \(|x - 4|, |y - 6|, |z - 12| \) each less than \(1/\sqrt{3} \)?

8. Assume an \( 8 \times 8 \) chessboard with the usual coloring. You may repaint all squares
   (a) of a row or column
   (b) of a \( 2 \times 2 \) square.
   The goal is to attain just one black square. Can you reach the goal?

9. There are \( a \) white, \( b \) black, and \( c \) red chips on a table. In one step, you may choose two chips of different colors and replace them by a chip of the third color. If just one chip will remain at the end, its color will not depend on the evolution of the game. When can this final state be reached?

10. Each of the numbers 1 to \( 10^6 \) is repeatedly replaced by its digital sum until we reach \( 10^6 \) one-digit numbers. Will these have more 1's or 2's?

11. An algorithm is defined as follows:
    Start: \((x_0, y_0)\) with \(0 < x_0 < y_0\).
    Step: \( x_{n+1} = \frac{x_n + y_n}{2}, \quad y_{n+1} = \sqrt{x_{n+1}y_n}. \)
    Prove that a common limit \( \lim_{n \to \infty} x_n = \lim_{n \to \infty} y_n \) exists and find it.

12. To each vertex of a pentagon, we assign an integer \( x_i \) with sum \( s = \sum x_i > 0 \). If \( x, y, z \) are the numbers assigned to three successive vertices and if \( y < 0 \), then we replace \((x, y, z)\) by \((x + y, -y, y + z)\). This step is repeated as long as there is a \( y < 0 \). Decide if the algorithm always stops.