HOMEWORK 0
DUE: 9/12

Read the first 7 pages of Hatcher’s notes (http://www.math.cornell.edu/~hatcher/TN/TNch0.pdf); we did this in class. Read further if you like; the rest of these notes generalize to Pythagorean quadruples, rational points on other quadratic curves, etc. Please do the following exercises (from Hatcher).

Problem 1.
(a) Make a list of the first 16 primitive Pythagorean triples \((a, b, c)\) with \(c \leq 100\) regarding \((a, b, c)\) and \((b, a, c)\) as the same triple.
(b) How many more would there be if we allowed nonprimitive triples?
(c) How many triples (primitive or not) are there with \(c = 65\)?

Problem 2. Find a right triangle whose sides have integer lengths and whose acute angles are close to 30 and 60 degrees by first finding an irrational value of \(t\) (Hatcher uses \(r\)) that corresponds to a right triangle with acute angles exactly 30 and 60 degrees, then choosing a rational number close to this irrational value of \(t\).

Problem 3. Find an infinite sequence of Pythagorean triples where two of the numbers in each triple differ by 2.

Problem 4 (optional). Show that for every Pythagorean triple \((a, b, c)\) the product \(abc\) must be divisible by 60. (It suffices to show that \(abc\) is divisible by 3, 4, and 5).

Also, please answer the following questions in an email to me as soon as possible so I have your email address.

Question 0. What is your name?

Question 1. What is your year/school affiliation?

Question 2. What is your major?

Question 3. What is the highest level math class you have taken or are taking concurrently?

Question 4. Why are you interested in number theory?

Question 5. Which of the following words/phrases/symbols are you comfortable with in the mathematical context: implies, \(\Rightarrow\), if and only if, \(\Leftrightarrow\), induction, contrapositive, convergence. (Don’t worry if your answer is none; I’m just trying to gauge the comfort level of the class)