Score:

Name:

Calculus II, Summer 2006
Final Exam

Instructions

1. Write all solutions in the space provided, and use the back pages if you have to.
2. For full credit you must show all work, including all substitutions and integration by parts that are performed. Providing only an answer will result in very few marks.
3. You may use calculators for doing numerical computations and graphing only. If you have a fancy calculator you can't use it for doing integrals or derivatives.
4. If you don't understand the wording of any of the questions or what exactly they're asking just ask me, but I won't give help on how to answer the questions.

1. (20 pts.) Solve the following differential equations:

   (a)
   $$y' = \frac{\ln x}{xy + xy^3}$$

   (b)
   $$y' + \frac{2}{x}y = \frac{y^3}{x^2}$$

2. (20 pts.) Solve the following differential equations:

(a)

$$2xy^3 + 3x^2y^2y' = 0$$

(b)

$$y' = \frac{x - y}{x}$$

3. (20 pts.) Find the interval of convergence of the following series:

(a)

$$\sum_{n=0}^{\infty} \frac{n}{4^n}(2x)^n$$

(b)

$$\sum_{n=0}^{\infty} \frac{(-1)^n}{n(n+1)} x^n$$

4. (20 pts.) Find the power series representation of the function. Then state its interval of convergence

(a)

$$f(x) = \frac{1}{16 + x^4}$$

(b)

$$g(x) = \frac{1}{(1 + x)^2}$$

Hint for both: Recall the power series expansion of $(1 + x)^{-1}$*, or in other words the formula for the sum of a geometric series. Both questions are related to that somehow.*

5. (20 pts.) Express the given power series in terms of common functions like $e^x, \sin x, \cos x$, etc.

(a)

$$\sum_{n=0}^{\infty} \frac{x^n}{2^n (n+1)!}$$

(b)

$$\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{4^{2n+1} (2n+1)!}$$

6. (20 pts.) Compute the following limits:

(a)

$$\lim_{x \to \infty} \left(1 + \frac{3}{x} + \frac{5}{x^3}\right)^x$$

(b)

$$\lim_{n \to \infty} n - \sqrt{n^2 - 1}$$

7. (20 pts.) Find the area inside the curve $r = 3 + 2\sin\theta$ for $0 \le \theta \le 2\pi$.

8. (10 pts.) Compute $(2+2i)^6$.

9. (20 pts.) You know the power series for $e^x, \sin\theta$ and $\cos\theta$. Plug in $x = i\theta$ into the power series for e^x, and use that to come up with Euler's formula

$$e^{i\theta} = \cos\theta + i\sin\theta$$