Let \((X, d)\) denote a metric space and let \(\mathcal{C}\) denote a collection of metrics on \(X\) such that \(d' \in \mathcal{C}\) implies \(c \cdot d' \in \mathcal{C}\), for any real number \(c > 0\). Put

\[
\rho(d, \mathcal{C}) = \inf_{d' \in \mathcal{C}} \inf \{c \mid d' \leq d \leq c \cdot d'\}.
\]

Let \(\mathcal{L}\) denote the collection of metrics on \(X\) of the form, \(d'(x_1, x_2) = \|f(x_1) - f(x_2)\|_{L^1}\), for some map \(f : X \to L^1\) and let \(\mathcal{N}\) denote the collection of metrics \(d\) on \(X\) such that \((X, d^{\frac{1}{2}})\) embeds isometrically in \(L^2\). It is easy to verify that \(\mathcal{C} \subset \mathcal{N}\), and so, \(\rho(d, \mathcal{N}) \leq \rho(\mathcal{L})\). If \(X\) is finite with cardinality \(n\), it was shown by Bourgain that \(\rho(d, \mathcal{L}) = O(\log n)\), for any metric \(d\).

Although the problem of computing \(\rho(d, \mathcal{L})\) exactly is equivalent to various other fundamental problems for which there is believed to be no polynomial time algorithm, there is a polynomial time algorithm for computing \(\rho(d, \mathcal{N})\). Goemans and Linial conjectured that for some universal constant \(C > 0\), independent of \(n\),

\[
\rho(d, \mathcal{N}) \leq C \cdot \rho(d, \mathcal{L}).
\]

Their conjecture was refuted by Khot-Vishnoi (2005) who gave a sequence of examples for which the best \(C\) grows at least like a constant times \(\log \log n\). We will discuss a very different sequence based on the Heisenberg group, for which \(C\) grows at least like \((\log n)^a\), for some explicit \(a > 0\). This is joint work with Kleiner and Naor. It is an outgrowth of earlier work of Lee-Naor and Cheeger-Kleiner.