A Tale of Two Indices

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A Tale of Two Indices: An Overview

- In 1993, the CBOE introduced the volatility index (VIX):
  - The VIX is a widely followed benchmark for stock index volatility.
  - The VIX is also known as the investor fear gauge.
- On September 22, 2003, the CBOE revamped its definition of the VIX and back calculated the new VIX up to 1990.
  - The ticker for the old VIX was switched to VXO.
- On March 26, 2004, the CBOE launched a new Exchange, the CBOE Futures Exchange (CFE) and began trading futures on the new VIX.
- Some questions addressed in this talk include:
  - *What are the differences between the two indices?*
  - *How do VIX and VXO behave historically?*
  - *What is the best forecast of subsequent realized volatility?*

Difference in Definitions and Calculations

- **VXO**: An average over 8 near-the-money Black-Scholes implied volatilities at the two nearest maturities on S&P 100 index.
  - Essentially, an estimate of the one-month at-the-money implied volatility.

- **VIX**: An average of out-of-the-money option *prices* (Q) across *all available strikes* on the S&P 500 index:

$$\sigma^2 = \frac{2}{T}\sum_i \frac{\Delta K}{K_i^2} e^{rT} Q(K_i, T) - \frac{1}{T}\left[\frac{F}{K_0} - 1\right]^2. \tag{1}$$

  - Linearly interpolate (extrapolate) over the nearest two maturities to obtain a 30-day estimate, in volatility percentages.

$$VIX = 100\sqrt{\frac{365}{30}\left[T_1\sigma_1^2\frac{N_{T_2} - 30}{N_{T_2} - N_{T_1}} + T_2\sigma_2^2\frac{30 - N_{T_1}}{N_{T_2} - N_{T_1}}\right]}. \tag{2}$$

Theoretical Underpinnings of the Old VIX (VXO)

- Black-Scholes implied volatility.
- However, under more general settings, VXO is an accurate approximation of the *volatility swap rate*,

$$VolS \equiv \mathbb{E}_0^{\mathbb{Q}}\sqrt{RV} = ATMV + O(T^{\frac{3}{2}}). \tag{3}$$

- The payoff of a volatility swap contract:

Vol Swap Rate ($VolS$)	$\rightleftharpoons$	**Realized Volatility** ($\sqrt{RV}$)

- The difference is converted into a dollar amount based on a notional figure.
- The contract has zero market value at inception, so that:

$$VolS_T = \mathbb{E}_t^{\mathbb{Q}}[\sqrt{RV}_T].$$

Theoretical Underpinnings of the New VIX

- Under very general settings, the new VIX is an approximation of the *variance swap rate*,

$$VarS \equiv \mathbb{E}_0^{\mathbb{Q}} RV_T = e^{rT} \int_0^{\infty} \frac{2Q_0(K,T)}{K^2} dK + \varepsilon, \tag{4}$$

- The payoff of a variance swap contract:

Variance Swap Rate ($VarS$) $\rightleftharpoons$ **Realized Variance** (RV)

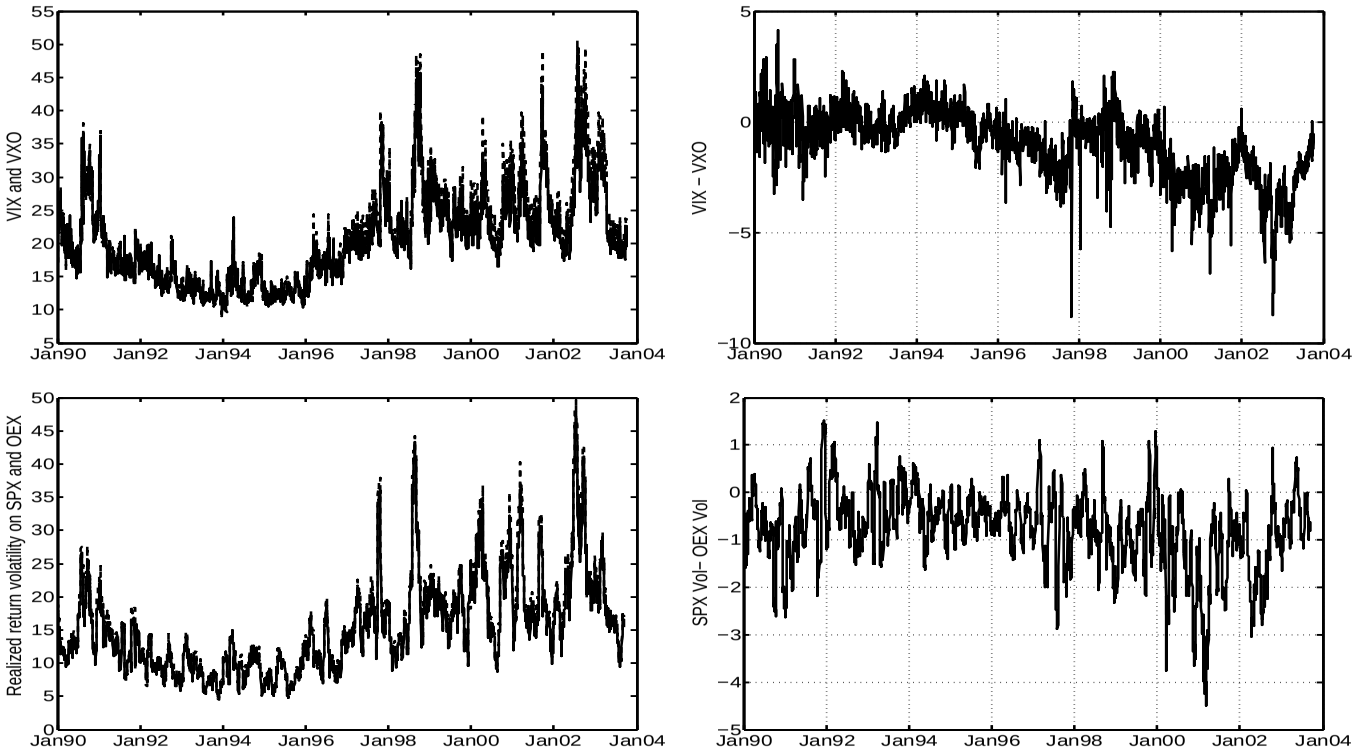
- The contract has zero market value at inception, so that

$$VarS_T = \mathbb{E}_t^{\mathbb{Q}}[RV_T].$$

Why Switch from VXO to VIX?

- The interpretation of VXO as an approximation to a volatility swap rate was not known until very recently.
- Even so, the volatility swap, which is the basis for the VXO, is much more difficult to hedge than the variance swap, which is the basis for the VIX.
- To hedge a variance swap, Neuberger/Dupire showed in the early 90's that we need to take
  - A static position in a portfolio of all out-of-the-money options:
  - A dynamic position in futures
- Since futures are costless, VIX can also simply be regarded as the value of a portfolio of options.

Time Series Behaviors

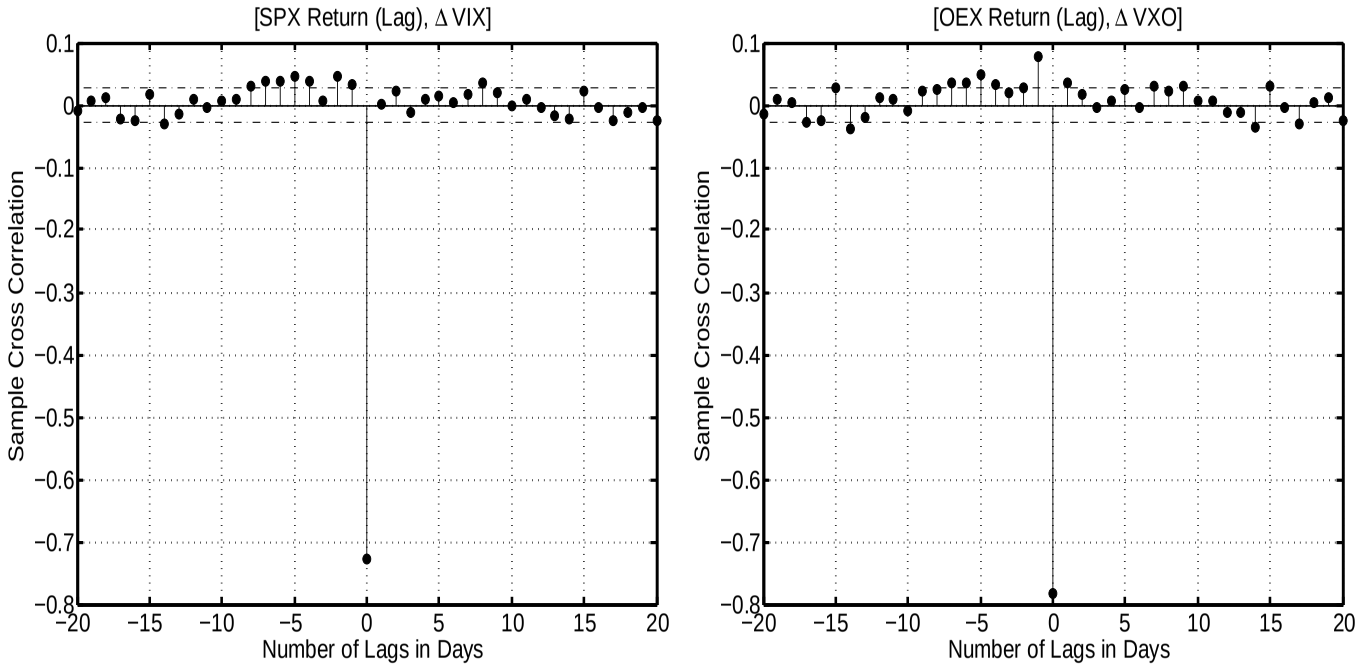


VIX and VXO follow each other closely; the difference in level reflects mainly the difference in the underlying stock index.

Summary Statistics

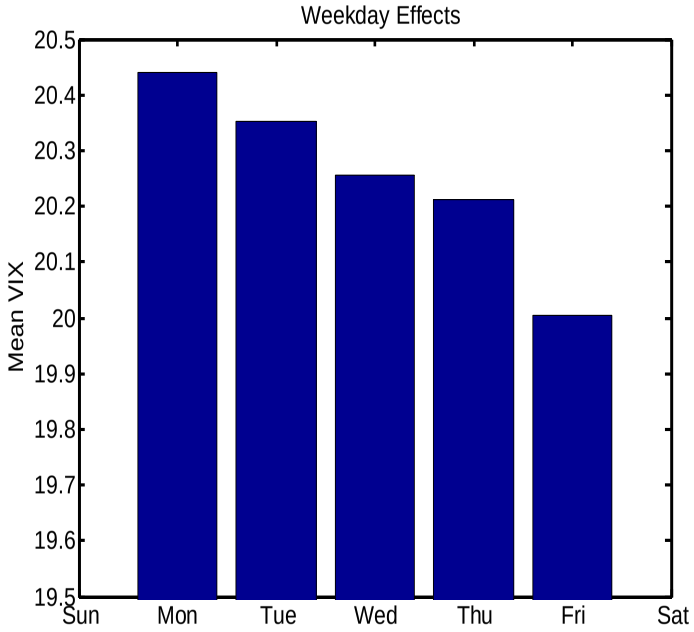
	VIX	SPX Vol	VXO	OEX Vol	VIX	SPX Vol	VXO	OEX Vol
		Levels				Daily Differences		
Mean	20.180	15.240	21.254	16.030	0.000	-0.001	0.001	-0.001
Stdev	6.486	7.087	7.391	7.527	1.060	0.863	1.216	0.907
Skew	0.807	1.282	0.811	1.257	0.668	0.728	0.676	0.540
Kurt	0.519	2.062	0.538	1.866	9.638	30.277	12.916	27.766
		Log Levels				Daily Log Differences		
Mean	2.955	2.625	2.998	2.674	0.000	-0.000	0.000	-0.000
Stdev	0.314	0.442	0.342	0.446	0.047	0.055	0.049	0.055
Skew	0.104	0.103	0.060	0.118	0.736	0.485	0.600	0.387
Kurt	-0.651	-0.382	-0.697	-0.424	6.729	17.833	7.157	18.543

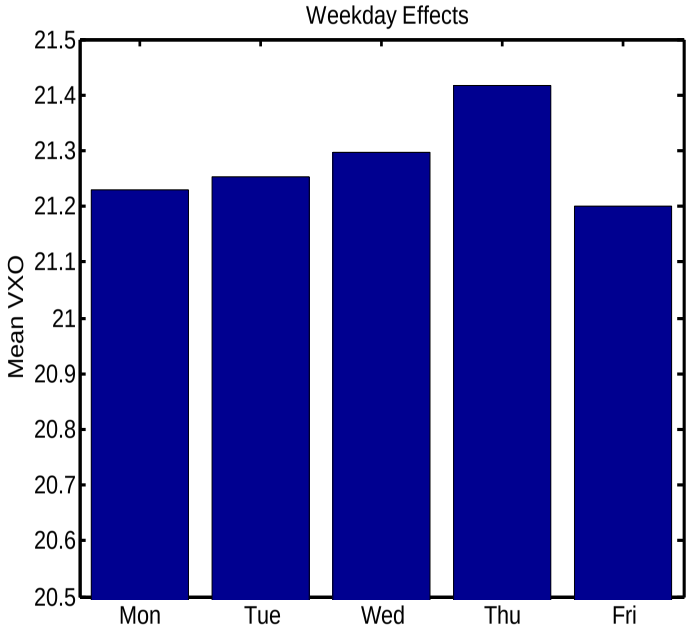
Cross-Correlation between Return and VIX(VXO)



Index returns forecast volatility, not the other way around.

Weekday Effects

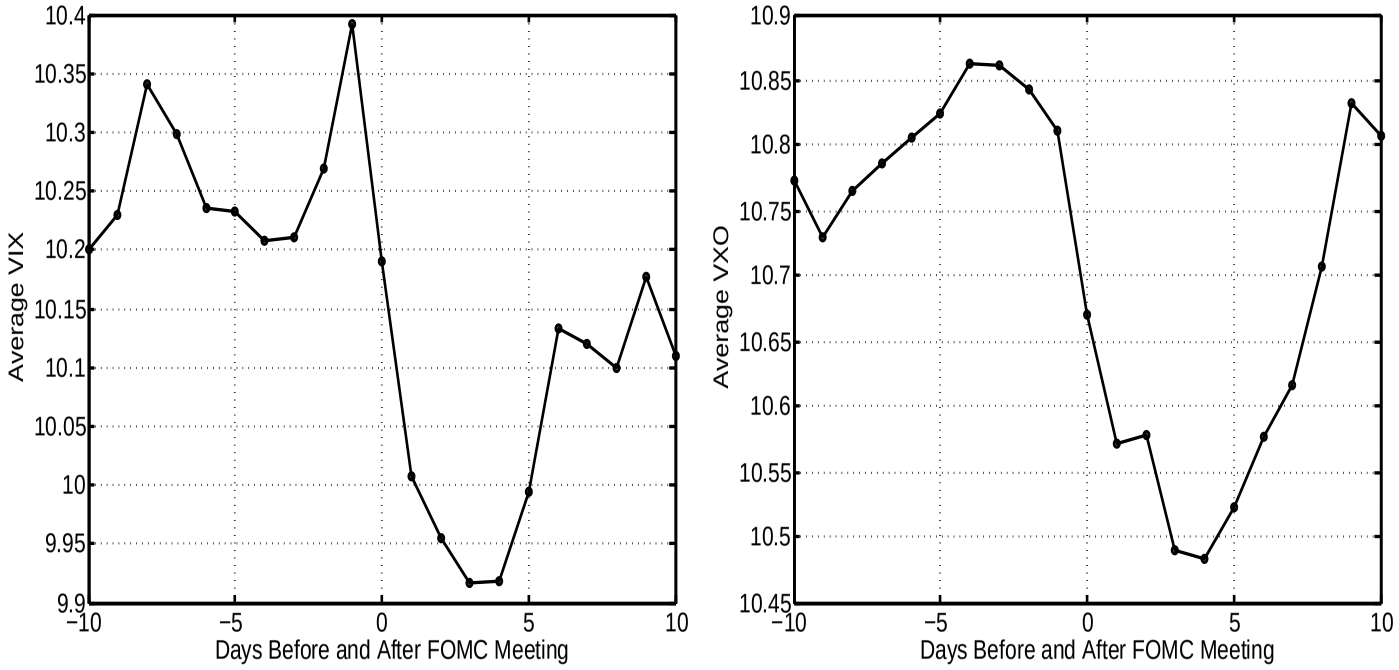




Friday average VIX is lower than averages on other weekdays.
VXO does not have this effect.

The FOMC Meeting Day Effect

Vol index levels 10 days before and after FOMC meeting days.



Volatility drops by about one percentage point after the meeting.

The Information Content

Series	Intercept		VIX/VXO		GARCH		R-square
			A. Forecasting $\ln \sqrt{RV}_{SPX}$				
$\ln VIX$	-0.660	(0.199)	1.112	(0.067)	—	—	0.621
GARCH	0.178	(0.166)	—	—	0.908	(0.061)	0.569
Joint	-0.541	(0.204)	0.813	(0.152)	0.284	(0.130)	0.633
			B. Forecasting $\ln RV_{OEX}$				
$\ln VXO$	-0.496	(0.179)	1.057	(0.059)	—	—	0.654
GARCH	0.171	(0.169)	—	—	0.912	(0.061)	0.572
Joint	-0.471	(0.182)	0.944	(0.149)	0.115	(0.137)	0.657

The vol indices are efficient forecasts of realized volatility;
GARCH vol is not needed once the index is included.