

Introducing the covariance swap

1998 focused our minds on correlation risk. Here, Peter Carr and Dilip Madan show how contracts paying the realised covariance between two currencies can be constructed by combining static option positions with continuous trading in underlying futures or forward contracts

Volatility swaps have recently emerged on several over-the-counter markets. These contracts pay the difference between the realised volatility over a specified time interval and a constant¹ agreed upon at the outset of the contract. The motivation for contracts whose payouts are tied to volatility has been discussed by several authors. For example, Gastineau (1977) and Galai (1979) propose the development of option indexes that can be used as the underlying for derivative contracts. Brenner & Galai (1989) propose the development of realised volatility indexes and futures and options contracts on these indexes. Similarly, Fleming, Ostdiek & Whaley (1993) describe the construction of an implied equity volatility index, while Whaley (1993) proposes derivative contracts written on this index. Grunbichler & Longstaff (1993) and Brenner & Galai (1993) develop a valuation model for options on volatility assuming a mean reverting volatility process.

In response to this, some volatility contracts have been listed. For example, the OMLX, the London-based subsidiary of the Swedish exchange OM, launched volatility futures at the end of 1996. In January 1998, Eurex launched its own futures based on its established implied volatility index. Thus far, the volume in these contracts has been disappointing.

One possible explanation for this outcome is that variance can already be traded by combining static positions in options on price with dynamic trading in the underlying. Neuberger (1990) showed that by delta-hedging a contract paying the log of the price, the hedging error accumulates to the difference between the realised variance and the fixed variance used in the delta-hedge. The contract paying the log of the price can be created with a static position in options, as shown in Breeden & Litzenberger (1978). Independently of Neuberger, Dupire (1993) showed that a calendar spread of two such log contracts has a payout that depends on the realised variance between the two maturities. Dupire (1996), Derman, Kani & Kamal (1997) and Derman & Kani (1998) showed how to create "local" volatility contracts that pay the realised variance at a particular time and price. Carr & Madan (1998) synthesise these developments in their review of volatility contracting. An important aspect of these developments is that the pricing and hedging of these variance contracts is completely insensitive to the choice of the volatility process. Just as the creation of a forward contract on a stock is independent of the stock price process, the creation of a contract paying the realised variance is independent of the stochastic volatility process.

Although variance contracts represent an important advance, one of the many issues bedeviling international investment in both financial and real assets is that both the variances and covariances of different currencies are generally unstable over time. This instability can hinder international expansion and can render mean variance analysis impractical for foreign in-

vestment. The objective of this paper is to demonstrate the creation of a covariance swap. This contract pays the excess of the realised covariance between two currencies over a constant specified at the outset of the contract. Such a contract would serve as a useful complement for the variance contracts that trade OTC on several currencies. By combining variance and covariance swaps, the realised variance of return on a portfolio of currencies can be locked in. Like the variance swap, the creation of a covariance swap is independent of the stochastic volatility process, as well as the stochastic correlation process.

Since the creation of the fixed leg of a covariance swap is trivial, we focus our attention on the creation of the floating leg. The development of a contract paying the realised covariance is a necessary precursor for the development of further derivatives written on covariance. Clearly, a calendar spread of two covariance contracts pays out the realised covariance between two future dates. In principle, the two future dates can be made arbitrarily close and an instantaneous forward price for covariance can be determined. As a consequence, one can develop a term structure of forward covariances and model its evolution through time. As with any forward price, the risk-neutral drift of this term structure would be zero. By dynamic trading in forex options and their underlying currencies, contracts with payouts that are non-linear in covariance can be constructed.

Our approach for creating a covariance contract assumes that we have a triangular market consisting of three exchange rates and European options written on each of the three rates. Margrabe (1993) studies triangular arbitrage in the forex market and Walter & Lopez (1998) imply out covariances using the Garman-Kohlhagen (1983) model. Of course, the Garman-Kohlhagen model assumes that volatilities and correlations are constant, so the use of implied Garman-Kohlhagen volatilities and correlations is theoretically unsound. In this paper, we show how to develop an alternative covariance forecast that uses option prices, but does not assume that volatilities and correlations are constant.

This paper is arranged as follows: the next section reviews the theory of static replication using options; the following section

reviews how static replication can be combined with the standard theory of dynamic replication using futures or forwards to create contracts written on realised variance; the subsequent section extends these results to the creation of covariance contracts; and the final section summarises and suggests some avenues for future research.

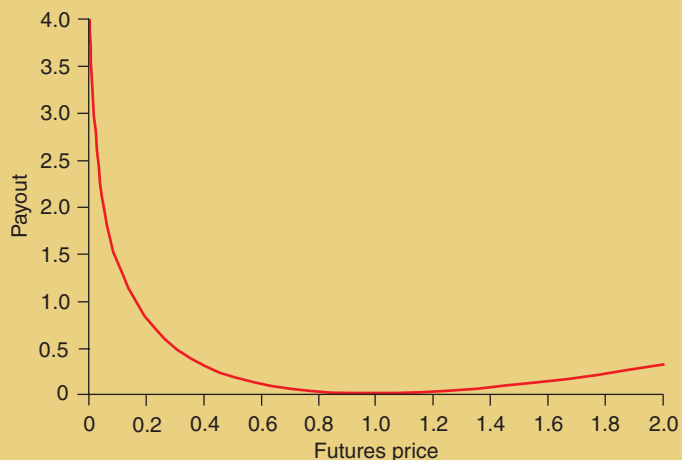
Review of static hedging using options

Here, we review the theory of static replication using options first developed in Ross (1976) and Breeden & Litzenberger (1978). Consider a sin-

One of the many issues bedeviling investment is that the variances and covariances of currencies are unstable over time

¹ This constant is often the implied volatility of an at-the-money option

Payout to delta-hedge at zero volatility to create contract paying realised variance ($F_0 = 1$)



gle period setting in which investments are made at time 0 with all payouts received at time T. In contrast to the standard intertemporal model, we assume that there are no trading opportunities other than at times 0 and T. We consider the replication of payouts at T that are a function of the futures forex rate or the forward forex rate for delivery at $T \geq 0$. We let F denote both the futures forex rate and the forward forex rate, although we recognise that the two can differ when interest rates are stochastic, as is assumed. We also assume that markets exist for European-style options written on this futures or forward price. This assumption appears restrictive since listed futures options are generally American-style and options written on forward prices are unusual. However, we note that by setting $T = T$, European-style options on the forward or futures exchange rate become European-style options on the spot exchange rate, which are available in both the listed and the OTC markets. To obtain exact results, we assume that there exists an entire continuum of positive strike rates. While the assumption of a continuum of strikes is far from standard, it is essentially the analogue of the standard assumption of continuous trading. Just as the latter assumption is frequently made as a reasonable approximation to an environment where investors can trade frequently, our assumption is a reasonable approximation when there are a large but finite number of option strikes.

It is widely recognised that this market structure allows investors to create any smooth function $f(F_T)$ of the terminal futures or forward forex rate F_T by taking a static position at time 0 in forex options.² When this theory is used to create variance and covariance contracts, this paper will show that it is only the second derivative of the payout that is relevant. Consequently, we will always choose f so that its value and slope vanish at the initial futures or forward forex rate F_0 (ie, $f(F_0) = f'(F_0) = 0$). The appendix shows that for this special case³, any twice differentiable payout can be spanned by the following position in out-of-the-money options:

$$f(F_T) = \int_0^{F_0} f''(K)(K - F_T)^+ dK + \int_{F_0}^{\infty} f''(K)(F_T - K)^+ dK \quad (1)$$

In words, to create a twice differentiable payout $f(\cdot)$ with value and slope

² This observation was first noted in Breeden & Litzenberger (1978) and established formally in Green & Jarrow (1987) and Nachman (1983)

³ See Carr & Madan (1997) for the corresponding result when the value and slope are unrestricted

⁴ We still require the existence of a unique non-negative weak solution to (3). If $\alpha_t F_t = a(F_t, t)$ and $\sigma_t F_t = b(F_t, t)$, then sufficient conditions for the existence of a unique weak solution are given in Stroock and Varadhan (1969). These are that the function $a(F, t)$ be measurable and bounded and that the function $b(F, t)$ be continuous, bounded and non-negative. To rule out negative futures/forward forex rates, we impose absorption at the origin, ie, $a(0, t) = b(0, t) = 0$, $t \in [0, T]$

vanishing at F_0 , buy $f''(K)dK$ puts at all strikes less than F_0 and buy $f''(K)dK$ calls at all strikes greater than F_0 .

In the absence of arbitrage, a decomposition similar to (1) must prevail among the initial values. Specifically, if we let V_0^f , $P_0(K)$ and $C_0(K)$ denote the initial prices of the payout $f(\cdot)$, the put and the call, respectively, then the no-arbitrage condition requires that:

$$V_0^f = \int_0^{F_0} f''(K)P_0(K)dK + \int_{F_0}^{\infty} f''(K)C_0(K)dK \quad (2)$$

Thus, the value of an arbitrary payout can be obtained from the option prices. Note that no assumptions were made regarding interest rates or the stochastic process governing the futures or forward forex rate.

Variance contracting

To create contracts on the realised variance of returns from investing in a currency, we now assume that interest rates are constant and that the underlying forward or futures price process is continuous. We further assume that investors can trade continuously and that forex futures contracts mark-to-market continuously. The process followed by the futures or forward forex rate is described by the following stochastic differential equation:

$$dF_t = \alpha_t F_t dt + \sigma_t F_t dW_t, \quad t \in [0, T] \quad (3)$$

where $F_0 > 0$, and α_t and σ_t are arbitrary processes denoting the relative drift and relative diffusion coefficients respectively. Thus, σ_t^2 is the variance rate of relative changes in the futures/forward forex rate and, like α_t , is an arbitrary⁴ unknown stochastic process. While one could specify a stochastic process for this variance rate and develop the correct delta-hedge in such a model, such an approach is subject to significant model risk since one is unlikely to guess the correct process. Furthermore, such models generally require dynamic trading in options, which is costly in practice. Consequently, in what follows we leave the process for the variance rate unspecified and restrict dynamic strategies to forward or futures contracts alone.

Let $f(F)$ be a twice differentiable function of the futures or forward forex rate F . By Itô's lemma:

$$f(F_T) = f(F_0) + \int_0^T f'(F_t) dF_t + \frac{1}{2} \int_0^T f''(F_t) \sigma_t^2 F_t^2 dt \quad (4)$$

Assuming that $f(F_0) = f'(F_0) = 0$ and solving for the last term gives:

$$\frac{1}{2} \int_0^T f''(F_t) \sigma_t^2 F_t^2 dt = f(F_T) - \int_0^T f'(F_t) dF_t \quad (5)$$

The left-hand side is a payout at T, which is based on the paths of both the realised instantaneous volatility σ_t^2 and the forex rate F_t . In contrast, the right-hand side of (5) depends only on the forex rate path. It results from combining the payout $f(F_T)$ obtained from a static position in options maturing at T with the payout from a dynamic strategy in either futures or forwards. If futures contracts are used, then the last term in (5) arises from holding $-e^{-r(T-t)} f'(F_t)$ futures contracts over the time interval $[0, T]$ (assuming continuous marking-to-market and that the margin account balance earns interest at the risk-free rate). If zero-cost forward contracts are used, then the last term in (5) arises from continuously rolling over $-f'(F_t)$ T-maturity forward contracts over the time interval $[0, T]$. In either case, the dynamic strategy can be interpreted as an attempt to create the payout $-f(F_T)$ at T, conducted under the false assumption of zero volatility. Since realised volatility will be positive, an error arises, and the magnitude of this error is given by:

$$\int_0^T \frac{F_t^2}{2} f''(F_t) \sigma_t^2 dt$$

which is the left-hand side of (5).

Now consider the following payout function $\phi(F)$ (see figure):

$$\phi(F_T) \equiv 2 \left[\frac{F_T}{F_0} - 1 - \ln \left(\frac{F_T}{F_0} \right) \right] \quad (6)$$

This payout is twice the difference between the discretely compounded return:

$$\frac{F_T}{F_0} - 1$$

and the continuously compounded return:

$$\ln \left(\frac{F_T}{F_0} \right)$$

The first derivative is given by:

$$\phi'(F) = 2 \left[\frac{1}{F_0} - \frac{1}{F} \right] \quad (7)$$

Thus, the value and slope both vanish at $F = F_0$ as required. The second derivative of ϕ is simply:

$$\phi''(F) = \frac{2}{F^2} \quad (8)$$

Substituting (6) to (8) into (5) results in a relationship between a contract paying the realised variance over the time interval $(0, T]$ and two payouts based on the forex rate:

$$\int_0^T \sigma_t^2 dt = 2 \left[\frac{F_T}{F_0} - 1 - \ln \left(\frac{F_T}{F_0} \right) \right] + 2 \int_0^T \left[\frac{1}{F_t} - \frac{1}{F_0} \right] dF_t \quad (9)$$

The first term on the right-hand side arises from a static position in options. Substituting (8) into (2) implies that the required position is given by:

$$2 \left[\frac{F_T}{F_0} - 1 - \ln \left(\frac{F_T}{F_0} \right) \right] = \int_0^{F_0} \frac{2}{K^2} (K - F_T)^+ dK + \int_{F_0}^{\infty} \frac{2}{K^2} (F_T - K)^+ dK \quad (10)$$

Thus, to create the contract paying $\int_0^T \sigma_t^2 dt$ at T , at $t = 0$, the investor should buy:

$$\frac{2}{K^2} dK$$

units of each out-of-the-money option maturing at T . The initial cost of this position is given by:

$$\int_0^{F_0} \frac{2}{K^2} P_0(K, T) dK + \int_{F_0}^{\infty} \frac{2}{K^2} C_0(K, T) dK \quad (11)$$

The investor should also start a dynamic strategy in futures or forward contracts. If futures are used, the investor holds:

$$2e^{-r(T-t)} \left[\frac{1}{F_t} - \frac{1}{F_0} \right]$$

futures contracts for each $t \in [0, T]$. If zero-cost forward contracts are used, the investor continuously rolls over:

$$2 \left[\frac{1}{F_t} - \frac{1}{F_0} \right]$$

T -maturity forward contracts for each $t \in [0, T]$.

The net payout at T is:

$$2 \left[\frac{F_T}{F_0} - 1 - \ln \left(\frac{F_T}{F_0} \right) \right] + 2 \int_0^T \left[\frac{1}{F_t} - \frac{1}{F_0} \right] dF_t = \int_0^T \sigma_t^2 dt$$

as required. Since the initial cost of achieving this payout is given by (11), an interesting forecast $\hat{\sigma}_{0,T}^2$ of the variance over $(0, T]$ is given by the future value of this cost:

$$\hat{\sigma}_{0,T}^2 = \int_0^{F_0} \frac{2e^{rT}}{K^2} P_0(K, T) dK + \int_{F_0}^{\infty} \frac{2e^{rT}}{K^2} C_0(K, T) dK$$

Appendix: spanning with options

The fundamental theorem of calculus implies that for any fixed F_0 :

$$\begin{aligned} f(F) &= f(F_0) + \mathbf{1}(F > F_0) \int_{F_0}^F f'(u) du - \mathbf{1}(F < F_0) \int_F^{F_0} f'(u) du \\ &= \mathbf{1}(F > F_0) \int_{F_0}^F \left[f'(F_0) + \int_{F_0}^u f''(v) dv \right] du \\ &\quad - \mathbf{1}(F < F_0) \int_F^{F_0} \left[f'(F_0) - \int_u^{F_0} f''(v) dv \right] du \end{aligned}$$

since $f(F_0) = 0$. Noting that $f'(F_0)$ does not depend on u and reversing the order of integration yields:

$$f(F) = f'(F_0)(F - F_0) + \mathbf{1}(F > F_0) \int_{F_0}^F \int_{F_0}^u f''(v) dudv + \mathbf{1}(F < F_0) \int_F^{F_0} \int_F^v f''(v) dudv$$

Setting $f'(F_0) = 0$ and performing the integral over u yields:

$$f(F) = \mathbf{1}(F > F_0) \int_{F_0}^F f''(v)(F - v) dv + \mathbf{1}(F < F_0) \int_F^{F_0} f''(v)(v - F) dv$$

and this may be equivalently rewritten as:

$$f(F) = \int_{F_0}^{\infty} f''(v)(F - v)^+ dv + \int_0^{F_0} f''(v)(v - F)^+ dv$$

■

In contrast to implied volatility, this forecast does not use a model in which volatility is assumed to be constant. However, in common with any forward price, this forecast is a reflection of both statistical expected value and risk aversion. Consequently, by comparing this forecast with the ex-post outcome, the market price of volatility risk can be inferred. This forecast is also the fixed rate in a variance swap implied by the absence of arbitrage.

Covariance contracting

The objective of this section is to create a contract paying the realised covariance of returns between two exchange rates, where both rates have the same base currency. We denote the two exchange rates used in the covariance calculation by $F^{a/b}$ and $F^{c/b}$. We assume there exist European-style forex options written on the terminal futures or forward exchange rate for all three possible pairings of the three currencies, $F_T^{a/c}$, $F_T^{a/b}$ and $F_T^{c/b}$. Absence of triangular arbitrage forces the following relationship at all times t prior to maturity:

$$F_t^{a/c} = F_t^{a/b} / F_t^{c/b}, t \in [0, T] \quad (12)$$

Setting $t = T$ and taking logs implies:

$$\ln F_T^{a/c} = \ln F_T^{a/b} - \ln F_T^{c/b}$$

Similarly, setting $t = 0$ in (12) and taking logs implies:

$$\ln F_0^{a/c} = \ln F_0^{a/b} - \ln F_0^{c/b}$$

Subtracting equations gives a relationship among continuously compounded returns:

$$\ln \left(\frac{F_T^{a/c}}{F_0^{a/c}} \right) = \ln \left(\frac{F_T^{a/b}}{F_0^{a/b}} \right) - \ln \left(\frac{F_T^{c/b}}{F_0^{c/b}} \right)$$

Taking the variance of both sides gives:

$$\int_0^T \sigma_{t,a/c}^2 dt = \int_0^T \sigma_{t,a/b}^2 dt + \int_0^T \sigma_{t,c/b}^2 dt - 2 \int_0^T \gamma_{t,a/b,c/b} dt$$

where $\gamma_{t,a/b,c/b}$ denotes the realised covariance of returns between forex rates $F^{a/b}$ and $F^{c/b}$. Solving for the covariance term gives:

$$\int_0^T \gamma_{t,a/b,c/b} dt = \frac{1}{2} \int_0^T \sigma_{t,a/b}^2 dt + \frac{1}{2} \int_0^T \sigma_{t,c/b}^2 dt - \frac{1}{2} \int_0^T \sigma_{t,a/c}^2 dt$$

From the results of the previous section, each integral can be created via a static position in options and dynamic trading in the underlying:

$$\begin{aligned} \int_0^T \gamma_{t,a/b,c/b} dt &= \frac{F_T^{a/b}}{F_0^{a/b}} - 1 - \ln\left(\frac{F_T^{a/b}}{F_0^{a/b}}\right) + \int_0^T \left[\frac{1}{F_t^{a/b}} - \frac{1}{F_0^{a/b}} \right] dF_t^{a/b} \\ &\quad \frac{F_T^{c/b}}{F_0^{c/b}} - 1 - \ln\left(\frac{F_T^{c/b}}{F_0^{c/b}}\right) + \int_0^T \left[\frac{1}{F_t^{c/b}} - \frac{1}{F_0^{c/b}} \right] dF_t^{c/b} \\ &\quad - \frac{F_T^{a/c}}{F_0^{a/c}} + 1 + \ln\left(\frac{F_T^{a/c}}{F_0^{a/c}}\right) - \int_0^T \left[\frac{1}{F_t^{a/c}} - \frac{1}{F_0^{a/c}} \right] dF_t^{a/c} \end{aligned} \quad (13)$$

The covariance contract is created via a static position in options on all three futures/forwards and by dynamic trading in all three underlying futures/forwards.

The initial cost of the static options position is given by:

$$\begin{aligned} &\int_0^{F_0^{a/b}} \frac{1}{K^2} P_0^{a/b}(K, T) dK + \int_{F_0^{a/b}}^{\infty} \frac{1}{K^2} C_0^{a/b}(K, T) dK \\ &+ \int_0^{F_0^{c/b}} \frac{1}{K^2} P_0^{c/b}(K, T) dK + \int_{F_0^{c/b}}^{\infty} \frac{1}{K^2} C_0^{c/b}(K, T) dK \\ &- \int_0^{F_0^{a/c}} \frac{1}{K^2} P_0^{a/c}(K, T) dK - \int_{F_0^{a/c}}^{\infty} \frac{1}{K^2} C_0^{a/c}(K, T) dK \end{aligned} \quad (14)$$

The investor should also conduct a dynamic strategy in either futures or forward contracts. If futures contracts are used, the investor holds:

$$e^{-r(T-t)} \left[\frac{1}{F_t^{a/b}} - \frac{1}{F_0^{a/b}} \right]$$

futures contracts on futures forex rate $F^{a/b}$,

$$e^{-r(T-t)} \left[\frac{1}{F_t^{c/b}} - \frac{1}{F_0^{c/b}} \right]$$

futures contracts on futures forex rate $F^{c/b}$ and

$$-e^{-r(T-t)} \left[\frac{1}{F_t^{a/c}} - \frac{1}{F_0^{a/c}} \right]$$

futures contracts on futures forex rate $F^{a/c}$ for each $t \in [0, T]$. If zero-cost forward contracts are used, the number of forwards held is obtained from the corresponding number of futures by suppressing the discount factor $e^{-r(T-t)}$. In either case, the net payout at T is given by either side of (13). Since the initial cost of achieving this payout is given by (14), an interesting forecast $\hat{\gamma}_{[0,T]}^2$ of the covariance between $F^{a/b}$ and $F^{c/b}$ over $(0, T]$ is given by the future value of this cost:

$$\begin{aligned} \hat{\gamma}_{[0,T]}^2 &= e^{rT} \left\{ \int_0^{F_0^{a/b}} \frac{1}{K^2} P_0^{a/b}(K, T) dK + \int_{F_0^{a/b}}^{\infty} \frac{1}{K^2} C_0^{a/b}(K, T) dK \right. \\ &\quad \left. + \int_0^{F_0^{c/b}} \frac{1}{K^2} P_0^{c/b}(K, T) dK + \int_{F_0^{c/b}}^{\infty} \frac{1}{K^2} C_0^{c/b}(K, T) dK \right. \\ &\quad \left. - \int_0^{F_0^{a/c}} \frac{1}{K^2} P_0^{a/c}(K, T) dK - \int_{F_0^{a/c}}^{\infty} \frac{1}{K^2} C_0^{a/c}(K, T) dK \right\} \end{aligned} \quad (15)$$

In contrast to implied covariance, this forecast does not use a model in which covariance is assumed to be constant. However, in common with any forward price, this forecast is a reflection of both statistical expected value and risk aversion. Consequently, by comparing this forecast with the outcome, the market price of covariance risk can be inferred. This forecast is also the fixed rate in a covariance swap implied by the absence of arbitrage.

Summary and suggestions for research

We show that by combining static positions in options with dynamic trading in futures or forwards, investors can synthesise contracts paying the realised variance of a currency or paying the realised covariance between two currencies. Importantly, these contracts were created without speci-

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fying either a volatility process or a correlation process.

By dynamically trading in at-the-money short-term forex options, one should be able to synthesise the payouts from options on realised variance or covariance. It would also be interesting to consider contracts whose payouts depend on the exchange rate. One can also consider using other approaches for creating covariance contracts, such as quantos options or spread options. Finally, it would be interesting to develop contracts on other statistics of the sample path, such as the standard deviation, the correlation, the Sharpe ratio or the realised skewness. ■

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