Courant Institute of Mathematical Sciences
Mathematics Colloquium
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Title: The Kolmogorov-Obukhov Theory of Turbulence

Abstract:

The Kolmogorov-Obukhov statistical theory of turbulence, with intermittency corrections, is derived from a stochastic Navier-Stokes equation with generic noise.

In 1941 Kolmogorov and Obukhov proposed a statistical theory of turbulence based on dimensional arguments. The main consequence and test of this theory was that the structure functions of the velocity differences of a turbulent fluid

$$E(|u(x,t)-u(x+l,t)|^p) = S_p = C_p l^{(p/3)}$$

should scale with the distance (lag variable) l between them, to the power p/3. This theory was immediately criticized by Landau for not taking into account the influence of the large flow structure on the constants Cp and later for not including the influence of the intermittency in the velocity fluctuations on the scaling exponents.

In 1962 Kolmogorov and Obukhov proposed a corrected theory were both of those issues were addressed. They also pointed out that the scaling exponents for the first two structure functions could be corrected by log-normal processes. For higher order structure functions the log-normal processes gave intermittency corrections inconsistent with contemporary simulations and experiments.

In this talk we discuss how the Central Limit Theorem and Large Deviation Principle produce additive noise that must be added to the Navier-Stokes equation for a proper description of fully developed turbulence. In addition, jumps in the velocity gradients produce multiplicative noise that must also be added to the deterministic equation. Following Hopf's work in 1952, we use this stochastic Navier-Stokes equation to compute the scaling of the structure functions and compute the PDF (probability distribution function) of the velocity differences. The Feynmann-Kac formula produces log-Poisson processes from the stochastic Navier-Stokes equation. These processes, first found by She, Leveque, Waymire and Dubrulle in 1995, give the correct intermittency corrections to the structure functions of turbulence.

The probability density function (PDF) of the velocity differences (two-point statistics) turns out to be the generalized hyperbolic distribution first suggested by Barndorff-Nilsen in 1977. We compare the theoretical PDF with PDFs obtained from DNS simulations and wind-tunnel experiments.