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Title: "Reformulation of the Covering and Quantizer Problems as Ground States of Interacting Particles"

I reformulate the covering and quantizer problems, well-known problems in discrete geometry [1], as the determination of the ground states of interacting particles in  $d$ -dimensional Euclidean space that generally involve single-body, two-body, three-body, and higher-body interactions [2]. This is done by linking the covering and quantizer problems to certain optimization problems involving the "void" nearest-neighbor functions that arise in the theory of random media and statistical mechanics. These reformulations, which again exemplifies the deep interplay between geometry and physics, allow one now to employ optimization techniques to analyze and solve these energy minimization problems. The covering and quantizer problems have relevance in numerous applications, including wireless communication network layouts, the search of high-dimensional data parameter spaces, stereotactic radiation therapy, data compression, digital communications, meshing of space for numerical analysis, and coding and cryptography, among other examples. The connections between the covering and quantizer problems and the sphere-packing and number-variance problems (related to problems in number theory) are discussed. I also show that disordered saturated sphere packings provide relatively thin (economical) coverings and may yield thinner coverings than the best known lattice coverings in sufficiently large dimensions. I derive improved upper bounds on the quantizer error using sphere-packing solutions, which are generally substantially sharper than an existing upper bound in low to moderately large dimensions. I demonstrate that disordered saturated sphere packings yield relatively good quantizers. Finally, I remark on possible applications of the results to the detection of gravitational waves.

1. J. H. Conway and N. J. A. Sloane, "Sphere Packings, Lattices and Groups," (Springer, New York, 1998).
2. S. Torquato, Phys. Rev. E, 82, 056109 (2010).