

# Fritz John's ellipsoid and the geometry of high dimensional convex bodies

by  
Gilles Pisier  
Texas A&M University  
College Station, TX 77843, U. S. A.  
and  
Université Paris VI  
Equipe d'Analyse, Case 186, 75252  
Paris Cedex 05, France

**Abstract.** The goal of this survey talk is to describe the major impact that the discovery of the maximal volume ellipsoid (now called John's ellipsoid) and the associated John decomposition of the identity had on the study of convex bodies of arbitrarily high dimension. Among the resulting applications, we will describe the influential concept of volume ratio (Kashin, Szarek-Tomczak-Jaegermann), and will present Keith Ball's characterization of the cube as having maximal volume ratio using the Brascamp-Lieb inequalities together with the John decomposition of a convex body. We will also describe some of Franck Barthe's work on the Brascamp-Lieb inequalities and their reverse form. In connection with volume ratio, the notion of Kashin splitting will be discussed together with the ramifications connected to Dvoretzky's famous theorem on spherical sections (weak cotype 2, weak Hilbert, V. Milman's QS-theorem). We will also mention some of the open problems related to that splitting, either in the non-commutative direction or in the computer science one (desire for effective algorithms). The second part will survey the more recent developments that were prompted by the Kannan-Lovász-Simonovits influential paper on the computation of volume of convex bodies. We will describe the subsequent improvements made by Bourgain, Rudelson and more recently by Adamczak-Litvak-Pajor-Tomczak-Jaegermann on the empirical covariance matrix (JAMS 2009 and CRAS to appear), as well as recent work by Batson-Spielman-Srivastava.