Certified Reduced Basis Methods for Harmonic Wave Problems

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Abstract

The development and application of models of reduced computational complexity is used extensively throughout science and engineering to enable the fast/real-time modeling of complex systems for control, design, or prediction purposes. These models, while often successful and of undisputed value, are, however, often heuristic in nature and the validity and accuracy of the output is often unknown. This limits the predictive value of such models.

In this talk we will discuss recent and ongoing efforts to develop reduced basis methods for which one can develop a rigorous a posteriori theory. The approach aims at formulating reduced models for parameterized linear partial differential equations. We focus the attention on wave problems in general and Maxwell's equations in particular as these introduce some specific challenges.

We will outline the theoretical developments of certified reduced basis methods, discuss an offline-online approach to ensure computational efficiency, and emphasize how an error estimator can be exploited to construct an efficient basis at minimal computational off-line cost. We also discuss recent improvements on the efficiency of the computation of the lower bounds for the error, using an improved Successive Constraint Method. The discussion will draw on examples based both on differential and integral equations formulations.

The performance of the certified reduced basis model will be illustrated through several examples drawn from electromagnetics, including problems with antennas and scattering applications. These examples will clearly highlight the major advantages of the proposed approach as well as key open challenges in the current approach.

Time permitting we will extend the discussion to include problems with parameterized geometries and the introduction of reduced element methods to enable the efficient and accurate modeling of networks and geometrically complex configurations.

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