Let (X,d) denote a metric space and let $\mathcal{C}$ denote a collection of metrics on X such that $d' \in \mathcal{C}$ implies $c \cdot d' \in \mathcal{C}$, for any real number $c > 0$. Put

$$\rho(d,\mathcal{C}) = \inf_{d'\in\mathcal{C}} \inf\{c \,|\, d' \leq d \leq c \cdot d'\}\,.$$

Let $\mathcal{L}$ denote the collection of metrics on X of the form, $d'(x_1,x_2) = \|f(x_1) - f(x_2)\|_{L_1}$, for some map $f : X \to L^1$ and let $\mathcal{N}$ denote the collection of metrics $\underline{d}$ on X such that $(X, \underline{d}^{\frac{1}{2}})$ embeds isometrically in L^2. It is easy to verify that $\mathcal{C} \subset \mathcal{N}$, and so, $\rho(d,\mathcal{N}) \leq \rho(\mathcal{L})$. If X is finite with cardinality n, it was shown by Bourgain that $\rho(d,\mathcal{L}) = O(\log n)$, for any metric d.

Although the problem of computing $\rho(d,\mathcal{L})$ exactly is equivalent to various other fundamental problems for which there is believed to be no polynomial time algorithm, there is a polynomial time algorithm for computing $\rho(d,\mathcal{N})$. Goemans and Linial conjectured that for some universal constant $C > 0$, independent of n,

$$\rho(d,\mathcal{N}) \leq C \cdot \rho(d,\mathcal{L}).$$

Their conjecture was refuted by Khot-Vishnoi (2005) who gave a sequence of examples for which the best C grows at least like a constant times $\log\log n$. We will discuss a very different sequence based on the Heisenberg group, for which C grows at least like $(\log n)^a$, for some explicit $a > 0$. This is joint work with Kleiner and Naor. It is an outgrowth of earlier work of Lee-Naor and Cheeger-Kleiner.