In the sequel you’ll find a list of the questions that (I could remember) I was asked in the general session of my oral exam. My topics were Real Variables, Complex Variables, ODE, PDE and Hilbert Spaces (no questions were asked about Hilbert Spaces, since we ran out of time).

**Real Variables**

Do you know an example of an uncountable set with zero measure? [Cantor set.] How do you know that it has zero measure? How do you know that it is uncountable? Would it make any difference if, instead of the middle third, the middle fifth of each interval were dropped at each step? [No.]

What does the Dominated Convergence Theorem state? Give a counter example for the case when the dominated hypothesis is violated.

What is the Fourier Series of a function? [Hilbert-space basis for the space $L^2([0,2\pi])$.] What is the inner product in this space? How do you show that the closure of the span is actually the whole space? What kind of convergence does one have? [From Hilbert space theory, $L^2$ convergence. But indeed also pointwise at points of continuity of $f$.] What about the end-points of the interval, if $f$ is smooth but $f(0) \neq f(2\pi)$? [Convergence to $\frac{f(0)+f(2\pi)}{2}$.]

**Complex Variables**

What is your favourite theorem in Complex Variables? [Cauchy-Goursat.] Can you prove it? [As in Ahlfors, subdividing the rectangle into smaller ones.]

Consider a function $f$ given by

$$f(z) = \int_{C_1} \frac{g(\xi)}{\xi - z} d\xi.$$
Is \( f \) entire? [No. It is analytic inside and outside the circle, though.] And when would such an \( f \) be entire? [Silence...] Assume \( f \) is entire. What is the value of \( f \) when \( z \) goes to infinity? [Zero.] So? [So \( f \) must be a constant (by Liouville’s thm.) and this constant must be zero.]

Can you prove the fundamental theorem of algebra? [I showed that there must be a root.] How do you know there are exactly \( n \) roots? [I quoted an elementary result in abstract algebra, that says that once one finds a root \( z_0 \), one can factor out \( (z - z_0) \).

**Ordinary Differential Equations**

What is *Phase Plane*?

What is the definition of stability? Consider an autonomous system. How would you find out if a critical point is stable? Discuss stability of a linear autonomous system. [Real part of eigenvalues.] Draw the orbits for the following cases in \( \mathbb{R}^2 \):

- \( \lambda_1 = 1, \lambda_2 = 1 \);
- \( \lambda_1 = -1, \lambda_2 = -1 \);
- \( \lambda_1 = 1, \lambda_2 = -1 \);
- \( \lambda_1 = i, \lambda_2 = -i \);
- \( \lambda_1 = i + c, \lambda_2 = -i + c, c \in (0, \infty) \);
- \( \lambda_1 = i - c, \lambda_2 = -i - c, c \in (0, \infty) \).

**Partial Differential Equations**

What is the heat equation? How do you solve it in the whole space?

What about the regularity of the solution of Poisson’s equation? [If the source term is \( L^2 \), then the solution is \( H^2 \).] What if the source term is \( C^0 \), is the solution \( C^2 \)? [No. But if the source term is \( C^{0,\gamma} \), then the solution is \( C^{2,\gamma} \).]