

**RANDOM COEFFICIENT H MODE  
CONFINEMENT SCALINGS**

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**Abstract**

The random coefficient two stage regression algorithm with the collisional Maxwell Vlasov constraint is applied to the I.T.E.R. H mode confinement database. The resulting scaling is similar to ITER89P with a slightly stronger size dependence.

## I. INTRODUCTION

The H mode confinement group has assembled an excellent H mode global energy confinement database<sup>1</sup>. In this letter, we apply the advanced statistical techniques, specifically the dimensionally constrained random coefficient (R.C.) regression of Refs. [2-4].

Our analysis differs from the H mode database group in several significant ways. First, we apply more stringent time stationarity and reactor relevant selection criteria. We also remove a number of influential outliers. The resulting single device scalings are more uniform and closer to standard L mode confinement scaling.

To model this tokamak to tokamak variation, we treat the scaling differences between devices as random variables. We begin by estimating a scaling expression using the random coefficient two step regression procedure of Refs. [2,3]. The collisional Maxwell Vlasov (C.M.V.) similarity is then tested and imposed<sup>4</sup>.

The ITER H mode database consists of one large tokamak, JET, one medium size tokamak, D3D and four small tokamaks, ASDEX, PDX, PBX, and JFT2-M. Thus the H mode database is significantly more statistically unbalanced than the corresponding L mode database.

Since the L mode database consists of roughly equal numbers of large and small tokamaks, the L mode size scaling is more accurately determinable. The H mode dataset includes only one large device, and thus it is also impossible to assess if the observed confinement dependencies on JET are the systematic differences from smaller experiments or are purely the manifestation of tokamak to tokamak differences. Furthermore, JET has a significantly

lower power density and beta value relative to the Troyon beta limit<sup>5</sup>.

In evaluating the confinement time scaling, we need to decide which confinement time to use. Most but not all devices provided both a magnetohydrodynamic (MHD) and a diamagnetic confinement time. The H mode confinement group recommended that the MHD confinement time be used for JFT2-M, D3D, PDX, and PBX-M, that the diamagnetic confinement time be used for JET and that an MHD equivalent diamagnetic confinement time be used for ASDEX.

We follow the H mode database group's recommendations except that for ASDEX, we use the geometric average of the diamagnetic confinement time and the MHD equivalent confinement time. The MHD confinement time is notoriously difficult to determine in circular crosssection devices. In addition, the principal ASDEX group member disapproves of the MHD correction advocated by the group as a whole. Finally, the MHD confinement time is almost always larger than the diamagnetic confinement time. By using the average of the diamagnetic confinement time and the MHD confinement time for ASDEX and the MHD confinement time for most other machines, we are effectively penalising ASDEX. This small penalty may compensate in some way for ASDEX's closed divertor.

In the H mode database, each experimental group calculates their own thermal confinement time. In our initial analysis, it appears that the systematic differences in the evaluation procedures are sufficiently large to make the determination of a combined thermal confinement time infeasible. The overall tendency appears to be to multiply the total confinement time by  $\bar{n}^2$ .

We assume that the isotope enhancement factor is  $M^{1/2}$ . Unfortunately,

no accurate measurement of the species mixture is available. We adopt the standard convention that  $H \rightarrow D$  discharges have isotope equal 1.5. The within tokamak isotope scaling for D3D is roughly  $M^{.56}$  and we correct the hydrogen D3D discharges by this amount. For PDX, the isotope scaling is roughly linear,  $M^{1.0}$ , which may be due to poor beam penetration. We therefore exclude all PDX  $H \rightarrow D$  discharges. JFT2-M has virtually no isotope dependence. Since the parametric dependencies in JFT2-M are nearly independent of isotope, we force the JFT2-M discharges to scale as  $M^{1/2}$ .

## II. INDIVIDUAL TOKAMAK CONFINEMENT SCALINGS

In their initial analysis of the H mode database, the H mode confinement group specified a standard data subset<sup>1</sup>. Their constraint consist of limits for the relative radiated power, the relative fast ion content, the relative time evolution of the plasma energy, the relative plasma beta and a lower bound on  $q_a$ . We accept all the standard constraints of H mode group. We impose the divertor pressure ratio constraint recommended by PDX.

In addition, we impose a number of additional constraints. Table 1 gives summary data for our data subset. Throughout this article, we describe the plasma current ,  $I_p$ , in units of MAmperes, the toroidal magnetic field,  $B_t$  in units of Teslas, the total heating power  $P$  in MWatts, and the line averaged plasma density in  $10^{19}$  particles per cubic meter.

We restrict our analysis to discharges with  $q_{95} \leq 6.5$ . For ASDEX, we restrict to discharges with  $q_{95} \leq 6.0$ .

In restricting the dataset to discharges where the auxiliary heating domi-

nates the Ohmic power, we must determine the Ohmic power. The H mode database contains  $P_{ohm}$  as measured by the instantaneous loop voltage. However the instantaneous loop voltage measures the edge toroidal electric field and not the core electric field. This discrepancy can be quite large for non-steady state plasmas. Furthermore, we want to restrict the ratio of the Ohmic power before auxiliary heating to the auxiliary power. The instantaneous Ohmic power is relevant for power balance calculations but not for our constraint. We note that almost all tokamaks observe an Ohmic electric field of approximately one Volt over a wide variety of Ohmic conditions. For the purposes of constraining the data, we define an equivalent Ohmic power,  $P_{ohm}^* \equiv I_p \times 1Volt$ .

*We restrict the relative Ohmic power by  $P_{ohm}^*/(P_{abs} + P_{ohm}) < .4$ . Since replacing hot plasma with more cold plasma is usually counterproductive, we require  $\dot{\bar{n}}\tau_E/\bar{n} < .4$ . We also eliminated half a dozen JET ELMy discharges at low values of  $\bar{n}/I_p$ .*

The time stationarity requirement on  $\bar{n}$  affects JET most strongly. The upper limit on  $q_a$  affects D3D most strongly. The requirement on the relative Ohmic power affects JFT2-M and JET more strongly. Imposing the JET Ohmic power restriction reduces the very strong  $\bar{n}$  and  $B_t$  scalings relative to the “standard” JET dataset.

In examining the residuals,  $y_i - \hat{y}_i(I_p, B_t, \bar{n}, P)$ , we noticed that in a number of cases the residual errors depended on the relative time change of the energy. In other words, the more nonstationary the discharge, the better the confinement. Therefore we imposed the stronger constraints of  $\dot{W}/(P_{abs} + P_{Ohm}) < .20$  for PBXM and  $\dot{W}/(P_{abs} + P_{Ohm}) < .165$  for PDX.

The D3D residual errors depend slightly on the normalised plasma energy change,  $\dot{W}$ . More precisely, the D3D discharges where  $\dot{W}$  is determinable systematically had better calculated confinement times than the D3D discharges where  $\dot{W}$  is indeterminable. In D3D,  $\dot{W}$  is almost always determinable in elm-free discharges and can seldom be evaluated in the ELMy D3D discharges. Thus systematically the elmfree D3D discharges received  $\dot{W}$  corrections in the energy confinement time and the elmy D3D discharges did not. The elm-free D3D discharges appear to have a slightly better calculated confinement but this may be an artifact of the  $\dot{W}$  analysis.

Aside from this slight confinement degradation in D3D, *no other tokamak shows any significant evidence of confinement differences between elmy and elmfree discharges.* We therefore combine the elmy and elmfree discharges.

The  $B_t$  dependence of confinement is extremely difficult to determine. To zeroth approximation, there is virtually no  $B_t$  variation in JFT2-M, PBX-M or PDX, little  $B_t$  variation in ASDEX, the edge  $q$ ,  $q_{95}$  is nearly fixed in D3D and to some extent in JET as well. The current and density scalings are coupled in D3D.

Furthermore, a number of influential outliers strongly impact the  $B_t$  scaling in PDX and ASDEX. To examine the  $B_t$  dependencies, we regressed the individual tokamaks versus,  $I_p$ ,  $\bar{n}$  and  $P$  and then plotted the residuals,  $y_i - \hat{y}_i(I_p, \bar{n}, P)$ , versus  $B_t$ .

For PDX, the vast majority of the PDX discharges show only a weak dependence on magnetic field. However several extremely low magnetic field discharges suffered from very degraded confinement. We assume that these ultra low  $B_t$  discharges lie outside the standard H mode parameter regime

and therefore remove these datapoints.

Similarly, ASDEX has one influential datapoint at extremely low magnetic field which performed extremely well and which was determining the ASDEX magnetic field scaling to be  $B_t^{-.193}$ . The rest of the ASDEX data indicated a weak to nonexistent  $B_t$  dependence. Therefore the ultra low  $B_t$  points were dropped.

Finally, JFT2-M has a small number (4) of very low  $B_t$  discharges relative to the mean JFT2-M magnetic field of 1.26 T. This group of discharges indicates that JFT2-M confinement has a small positive  $B_t$  exponent. Unfortunately, the JFT2-M is very unbalanced in the  $B_t$  covariate with almost all the data concentrated at 1.26T. Thus no accurate  $B_t$  scaling is possible for the JFT2-M dataset.

A comparison of Table 2a with Table 2b (corresponding to Table X of Ref. 1) shows that our within tokamak scalings vary significantly less in the new restricted dataset. Thus these restrictions result in a more uniform dataset which better characterises normal H mode discharges. Our scalings are almost always closer to a L mode type scaling than the standard subset of Ref. 1. We have scrutinised the more “pathological” scalings more carefully and therefore have tended to achieve this more uniform, L mode-like behavior.

Table 3 summarises how each of our constraints has reduced the number of discharges.

We agree with the H mode database group that the  $B_t$  scaling is poorly determined as a within tokamak covariate in the present database. Not only are the  $B_t$  scalings sensitive to the outlying datapoints, but also the root mean square error (RMSE) appears to be an extremely broad function of the

$B_t$  exponent. In cases such as this where the RMSE is much broader than the half width predicted by ordinary least squares (OLS) regression, the at least one of the assumptions of OLS regression, such as the correctness of the model or the independence of the errors, is almost always violated.

Furthermore, determining the standard between tokamak dependencies of  $R$ ,  $R/a$ ,  $\kappa$  and the overall constant is already a delicate and questionable procedure for six tokamaks. Treating  $B_t$  as an additional between tokamak covariate is clearly illposed in the present database.

In our analysis, we exclude the predominately  $B_t$  principal component in JFT2-M, PDX<sub>j</sub> and PBX-M. We also exclude the  $I_p$  principal component in PBX-M. We include all the D3D principal components, however the  $I_p$ ,  $B_t$ , and  $\bar{n}$  scalings are strongly coupled.

### III. RANDOM COEFFICIENT SCALING

We begin with an ordinary least square regression analysis of our 823 datapoint dataset <sup>1</sup> :

$$\tau_E M^{-1/2} = .06371 \left(\frac{R/a}{3.804}\right)^{-.217} \left(\frac{R}{1.696}\right)^{2.113} \left(\frac{\kappa}{1.398}\right)^{.379} \left(\frac{I_p}{.5667}\right)^{.729} \left(\frac{B_t}{1.774}\right)^{.511} \left(\frac{\bar{n}}{4.486}\right)^{.090} \left(\frac{P}{2.918}\right)^{-.510} . \quad (1)$$

The  $B_t$  scaling of  $B_t^{.511}$  even exceeds the  $B_t^{.291}$  exponent of JET. This can

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<sup>1</sup>We present our scalings centered about the database mean, thus the mean values of our database are apparent. Also if the scaling coefficients are rounded, the overall constant in the centered formulation does not need to be adjusted. The overall constant in the noncentered version should be corrected to match the overall constant of the centered formulation.

only happen when the within tokamak  $B_t$  scalings are poorly determined and the root mean squared error in fitting the corrected mean confinement time of the tokamaks to the between tokamak covariates,  $R$ ,  $R/a$ , and  $\kappa$  can be significantly reduced by including  $B_t$  as a basically between tokamak covariate.

The random coefficient within tokamak scalings are the matrix weighted average of the scalings of the individual tokamaks. Thus the  $B_t$  scaling will not exceed the maximum scaling observed in any tokamak.

We briefly summarise our random coefficient analysis<sup>2-4</sup>. First, for each tokamak, a scaling and covariance is estimated in  $I_p$ ,  $B_t$ ,  $\bar{n}$  and  $P$ . We calculate the empirical mean and covariance of these within tokamak scalings using the Swamy random coefficient weighting procedure. Second, the mean confinement time of each tokamak is corrected for the within tokamak scalings. The scalings with  $R/a$ ,  $\kappa$  and  $R$  are estimated by regressing the corrected mean energy times of the tokamaks. The error,  $\Sigma_{RC}$  in our estimate,  $\hat{\beta}_{RC}$ , of the scaling vector is given by Eq. (18a) of Ref. 2.

Since three of the tokamaks, JFT2M PDX, and PBX-M have virtually no  $B_t$  variation, we apply the projection missing value algorithm<sup>4</sup>. The projection missing value algorithm consists of using only the principal components of the within tokamak scalings which are estimatable. Unfortunately, *the uncertainty in the  $B_t$  scaling direction will be systematically underestimated since we are unable to compensate for the fewer degrees of freedom in the  $B_t$  direction.* In other words, when we estimate the covariance of the  $B_t$  exponents and divide through by the number of degrees of freedom, we use  $6 - 1 = 5$  instead of  $3 - 1 = 2$ .

*In the second stage regression, to determine the  $R$ ,  $R/a$  and  $\kappa$  scalings, we weight the larger tokamaks, D3D and JET, a factor of two larger than the smaller tokamaks. This big tokamak weighting factor is based on our subjective estimation of the relative importance bigger tokamaks should have in determining the scaling.*

In the second stage regression, we apply ridge regression with a relative ridge parameter of  $\theta_{nrm} = 0.005$ , a half percent downweighting. This downweighting affects the  $R$  exponent predominately.

$$\tau_E M^{-1/2} = .06298 \left( \frac{R/a}{3.804} \right)^{-.171} \left( \frac{R}{1.696} \right)^{1.988} \left( \frac{\kappa}{1.398} \right)^{.239} \left( \frac{I_p}{.5667} \right)^{.876} \left( \frac{B_t}{1.774} \right)^{.165} \left( \frac{\bar{n}}{4.486} \right)^{.075} \left( \frac{P}{2.918} \right)^{-.519} \quad (2)$$

The resulting scaling resembles the ITER89P scaling<sup>6</sup> except that it has a very strong size scaling. A principal components analysis of the random coefficients matrix,  $\Sigma_{RC}$ , reveals that the size scaling is the most poorly determined exponent. The large variance in the  $R$  exponent is a consequence of the database only containing one large tokamak.

The aspect ratio scaling uncertainty is relatively small due to the presence of PBX-M. The medium size tokamak, D3D, and the large tokamak, JET, in the database have small aspect ratios. Thus a negative exponent on the aspect ratio scaling indicates that D3D and JET have better confinement than a  $(R/a)^0$  dependence would indicate. If a  $(R/a)^0$  dependence were required in our scaling, an even stronger size dependence would result.

The scaling also has a noticeable component which violates collisional Maxwell Vlasov similarity.

#### IV. COLLISIONAL MAXWELL VLASOV CONSTRAINT

We would like to require that our log linear scaling expression be dimensionally consistent with the collisional Maxwell Vlasov (C.M.V.) system. Neglecting the ratio of the Debye length to all other scale lengths, the physical system is prescribed by three dimensionless variables<sup>7</sup>:  $\beta \equiv \bar{n}T_i/B_t^2$ ,  $\rho_{i*} \equiv (MT_i)^{1/2}/RB_t$ ,  $\nu_{i*} \equiv R\bar{n}q/T_i^2$ , together with the four naturally dimensionless variables:  $\kappa$ ,  $R/a$ ,  $q_{cyl}$  and  $M$ . As shown in Refs. [2,4,8], the requirement that  $\tau_E\Omega_i$  is a *log linear function of the dimensionless variables* can be treated as a linear constraint on the parameter vector,  $\vec{\beta}$ :  $\vec{\gamma} \cdot \vec{\beta}_{dl} + \gamma_B = 0$ .

Since the random coefficient algorithm not only produces efficient estimates of the parametric scalings but also a covariance matrix for the errors in the scaling, we are able to test this similarity ansatz.

Therefore we determine a C.M.V. constrained scaling within the multiple tokamak R.C. analysis. For the  $F(1, 5)$  distribution, the 50% confidence level (corresponding to the halfwidth) is  $T^2 = 0.528$ , the 75% confidence level is  $T^2 = 1.69$ , the 90% confidence level is  $T^2 = 4.06$ .

We find the test statistic,  $T^2 \equiv |\vec{\gamma} \cdot \hat{\vec{\beta}}_{RC} + \gamma_B|^2 / \vec{\gamma}^t \cdot \hat{\Sigma}_{RC} \cdot \vec{\gamma} = 1.99$ . Thus the null hypothesis that the data can be explained by a dimensionless power law scaling can be rejected with slightly more than a 75% certainty. This conclusion is based on our extremely crude but selfconsistent modeling of the R.C. covariance. In our L mode analysis, the corresponding result was  $T_{Lmode}^2 = 0.168$ . Thus the L mode data not only supported C.M.V. similarity, but also suggested that the L mode estimate of the dimensional projection,  $\vec{\gamma}^t \cdot \hat{\Sigma}_{RC} \cdot \vec{\gamma}$ , was far too large.

The dimensionally constrained H mode scaling is

$$\tau_E M^{-1/2} = .06301$$

$$\left(\frac{R/a}{3.804}\right)^{-.233} \left(\frac{R}{1.696}\right)^{1.577} \left(\frac{\kappa}{1.398}\right)^{.317} \left(\frac{I_p}{.5667}\right)^{.889} \left(\frac{B_t}{1.774}\right)^{.207} \left(\frac{\bar{n}}{4.486}\right)^{.105} \left(\frac{P}{2.918}\right)^{-.486} \quad (3)$$

We have determined the dimensionless scaling which is closest to the unconstrained R.C. scaling *measured in the  $\Sigma_{RC}^{-1}$  metric*. Since the dominant uncertainty occurs in the  $R$  exponent, our dimensionally constrained scaling differs from the unconstrained scaling of Eq. 2 by a weaker size scaling. Since  $R$  and  $R/a$  are strongly anticorrelated, the aspect ratio scaling decreases as well.

We give the scaling coefficients to three digits accuracy, not because of precision, but to reduce the extent which rounding error induces a violation of C.M.V. similarity. The noncentered version of the constrained scaling law is

$$\tau_E = .07904 M^{1/2} (R/a)^{-.233} R^{1.577} \kappa^{.317} I_p^{.889} B_t^{.207} \bar{n}^{.105} P^{-.486}$$

In accepting the C.M.V. constraint, we not only set the dimensional projection equal to zero, but also eliminate the R.C. variance in the dimensional direction from our uncertainty estimates. The projection of  $\Sigma_{RC}$  onto the dimensionless subspace,  $\Sigma_{dl}$ , satisfies  $\Sigma_{dl} = \Sigma_{RC} - \Sigma_{RC} \hat{\gamma} \hat{\gamma}^t \Sigma_{RC} / (\hat{\gamma}^t \Sigma_{RC} \hat{\gamma})$ .

To evaluate the statistical uncertainty in the predicted energy confinement for a given set of parameters, we transform the tokamak's parameters to the centered logarithmic variables,  $\vec{x}^t$ , and take the interproduct with the covariance matrix of Table 4. The centered  $\vec{x}^t$  variable is

$$\left( \left( \ln \frac{R}{a} - 1.336 \right), (\ln R - .528), (\ln \kappa - .335), 1, \right.$$

$$(\ln I_p + .568) , (\ln B_t - .573) , (\ln \bar{n} - 1.501) , (\ln P - 1.071)) .$$

The fourth index corresponds to the absolute constant in the scaling law.

For I.T.E.R., we assume the following parameter value:  $M = 2.5$ ,  $a = 2.15m$ ,  $R = 6.0m$ ,  $\kappa = 2.2$ ,  $I_p = 22MA$ ,  $B_t = 4.85T$ ,  $\bar{n} = 14.0 \times 10^{19}$ ,  $P_{\text{tot}} = 160MW$ . The resulting predicted confinement times is 4.65 sec with a statistical uncertainty factor of 32%.

For B.P.X., we use the following parameter values:  $M = 2.5$ ,  $a = .8m$ ,  $R = 2.59m$ ,  $\kappa = 2.2$ ,  $I_p = 11.8MA$ ,  $B_t = 9.0T$ ,  $\bar{n} = 40 \times 10^{19}$ ,  $P_{\text{tot}} = 80MW$ . We predict a B.P.X. H mode confinement time of 1.22 sec with an uncertainty factor of 26 %.

We note that the C.M.V. constraint reduces the estimated I.T.E.R. uncertainty significantly more than the estimated B.P.X. uncertainty. In the L mode database, size and magnetic field are strongly correlated, i.e. the larger experiments have large magnetic fields, especially TFTR and JT-60. JET has only a slightly larger magnetic field, and therefore differs mostly in size. Since the H mode database has only one large tokamak, the variance of the size exponent is crudely a factor of three larger than the L mode size exponent.

Our analysis of both the H and L mode databases underestimates the covariance of  $B_t$  exponent by setting the number of degrees of freedom equal to the number of tokamaks and not the number of tokamaks with  $B_t$  variation. However, the shrinkage factor is worse for the H mode database (2/5) than for the L mode database (6/10). Thus our random coefficient analysis finds that the size scaling is more uncertain than the magnetic field scaling.

This explains why we find that unconstrained B.P.X. has a much smaller unconstrained uncertainty than I.T.E.R.. The collisional Maxwell Vlasov constraint essentially couples the size and magnetic field scalings and therefore reduces the I.T.E.R. uncertainty more than the B.P.X. uncertainty.

## V. DISCUSSION

Global scaling expressions, in particular, the ITER-89P scaling<sup>6</sup>, have been successful in predicting the energy confinement in the new series of experiments. The dimensionally constrained R.C. scaling of Eq. 3 resembles the ITER-89P scaling in all parametric dependencies except that our H mode scaling has a slightly stronger size scaling.

The random coefficient model is applicable when the tokamak to tokamak differences are due to many small factors. If, however, this tokamak to tokamak variation is attributable to one or more important factors such as wall material or distance to the divertor plate, statistics is of little help in analyzing confinement.

We find a predicted I.T.E.R. confinement time of 4.65 sec with a statistical uncertainty of  $\pm 32\%$  and a predicted B.P.X. confinement time of 1.22 sec with a statistical uncertainty of  $\pm 26\%$ . The unaccounted for uncertainties are discussed in Ref. [3]. When the constraint of collisional Maxwell Vlasov similarity is imposed, the I.T.E.R. uncertainty is reduced from 34.9% to 32.4% while the B.P.X. uncertainty is slightly reduced from 27.9% to 26.3%.

We find that the H mode data has a much larger intrinsically dimensional component of the scaling than the corresponding L mode data. This may be

caused by the presence of other hidden variables.

We close on an optimistic note. We have repeated our constrained R.C. analysis, however in the second stage regression, we have weighted the larger tokamaks, D3D and JET even more heavily than the factor of two used in the present analysis. We find that the large tokamak weighted scaling has an even stronger size scaling and higher predicted confinement times for both I.T.E.R. and B.P.X. than Eq. 3. This may indicate favorable departures of the confinement time scaling from Eq. 3 for reactor size devices.

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