The Volatility Surface: Errata

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- **Page xxiii** "what it is that analysis of stock prices can tell **us** about how options ought to be priced".
- **Page 7** In the first equation, the coefficient of

$$S\,\frac{\partial V}{\partial S}$$

should be $+r$ not $-r$. Moreover dZ_2 in the fourth line of the equation should have a prefactor η. Thus the equation at the top of page 7 should now read:

$$\begin{aligned} & d\Pi_1 - r\,\Pi_1\,dt \\ = & \left\{\frac{\partial V}{\partial t} + \frac{1}{2}\,v\,S^2\,\frac{\partial^2 V}{\partial S^2} + \rho\,\eta\,v\,\beta\,S\,\frac{\partial^2 V}{\partial v\,\partial S} + \frac{1}{2}\eta^2\,v\,\beta^2\,\frac{\partial^2 V}{\partial v^2} + r\,S\,\frac{\partial V}{\partial S} - r\,V\right\}\,dt \\ + & \frac{\partial V}{\partial v}\,dv \\ = & \beta\,\sqrt{v}\,\frac{\partial V}{\partial v}\,\{\phi(S,v,t)\,dt + \eta\,dZ_2\} \end{aligned}$$

(Thanks to Jan Obłój of Imperial College)

- **Page 10** In the first equation, there should be no S in front of $\frac{\partial}{\partial S_T}$. The equation should read

$$\frac{1}{2}\frac{\partial^2}{\partial S_T^2}\left(\sigma^2 S_T^2\,\varphi\right) - \frac{\partial}{\partial S_T}\left(\mu\,S_T\,\varphi\right) = \frac{\partial \varphi}{\partial T}$$

Also, the equation lower down on page 10:

$$\begin{aligned}\frac{\partial C}{\partial T} &= \frac{\sigma^2 K^2}{2}\varphi + \int_K^\infty dS_T\,\mu\,S_T\,\varphi \\ &= \frac{\sigma^2 K^2}{2}\frac{\partial^2 C}{\partial K^2} + \mu\,(T)\,\left(-K\frac{\partial C}{\partial K}\right)\end{aligned}$$

should be corrected to:

$$\begin{aligned}\frac{\partial C}{\partial T} &= \frac{\sigma^2 K^2}{2}\varphi + \int_K^\infty dS_T\,\mu\,S_T\,\varphi \\ &= \frac{\sigma^2 K^2}{2}\frac{\partial^2 C}{\partial K^2} + \mu\,(T)\,\left(C-K\frac{\partial C}{\partial K}\right)\end{aligned}$$

(Thanks to Jan Obłój of Imperial College)

- **Page 12** The last sentence should read "equation (1.8) becomes" rather than "equation (1.4) becomes".
  (Thanks to Dan Pirjol of Markit)

- **Page 33** In equation (3.14), $\sqrt{v_t}$ should be inside the expectation. (3.14) would then read:

$$du_t \;=\; -\lambda(u_t-\bar{v})dt + \frac{\rho\eta}{2}u_t dt + \rho\eta\frac{x_T}{\hat{w}_T}d\hat{w}_t + \sqrt{1-\rho^2}\,\eta\,\mathbb{E}[\sqrt{v_t}\,dW_t\,|x_T]$$

  Also, the line just after equation (3.15) should read: "with $\hat{v}'_s := (v_0-\bar{v}')\,e^{-\lambda' s}+\bar{v}'$." (an extra subscript 0 on the v).

  On the last line of page 33, we should replace the words "Brownian Bridge" with the words "Brownian Bridge-like".
  (Thanks to Jan Obłój)

- **Page 35** Formula (3.19) should read:

$$\begin{aligned}\sigma_{BS}(K,T)^2 &\approx \frac{\hat{w}'_T}{T} + \rho\eta\frac{x_T}{T^2}\int_0^T dt\int_0^t e^{-\lambda'(t-s)}ds \\ &= \frac{\hat{w}'_T}{T} + \rho\eta\,\frac{x_T}{\lambda' T}\left\{1-\frac{\left(1-e^{-\lambda' T}\right)}{\lambda' T}\right\}\end{aligned}$$

  – the current version has an extra factor $\frac{1}{T}$ inside the first integral.

Also, further down on the same page there are three references to:

$$\frac{\partial}{\partial x_t}\sigma_{BS}(K,T)^2$$

These should of course read

$$\frac{\partial}{\partial x_T}\sigma_{BS}(K,T)^2$$

(Thanks to Paul Jones of Imperial College)

- **Page 44** Second paragraph, "from equation (3.11)" should read "from equation (3.15)".
(Thanks to Jan Obłój)

- **Page 49** "We note too that the Heston-**Nandi** model and its local volatility equivalent are single-factor".

- **Page 57** Line 5 should read "The characteristic function for **an** exponential".
(Thanks to Jan Obłój)

- **Page 61** There is a subscript BS missing in equation (5.9) which should read:

$$\begin{aligned} C_J(S,K,\Delta T) &\approx (1-\lambda\Delta T)\, C_{BS}(Se^{\mu_J \Delta T},K,\Delta T) + \lambda\Delta T\, C_{BS}(JS,K,\Delta T) \\ &= C_{BS}(Se^{\mu_J \Delta T},K,\Delta T) + O(\Delta T) \end{aligned}$$

(Thanks to Jan Obłój)

- **Page 68** Factor T missing in the denominator of the last equation on the page. It should read

$$\begin{aligned} I(u,T) &= \frac{1}{T}\int_0^T e^{\gamma_v D(u,t)}dt \\ &= -\frac{\mathbf{1}}{\mathbf{T}}\frac{2\gamma_v}{p_+p_-}\int_0^{-\gamma_v D(u,T)} \frac{e^{-z}dz}{(1+z/p_+)(1+z/p_-)} \end{aligned}$$

(Thanks to Roger Lord)

- **Page 89** 6 lines from the bottom, "earlier in Chapter 7" should read "earlier in Chapter 3".
  (Thanks to Jan Obłój)

- **Page 102** The expression for v_{loc} should read:

$$v_{loc} = \frac{\frac{\partial w}{\partial T}}{1 - \frac{k}{w}\frac{\partial w}{\partial k} + \frac{1}{4}\left(-\frac{1}{4} - \frac{1}{w} + \frac{k^2}{w^2}\right)\left(\frac{\partial w}{\partial k}\right)^2 + \frac{1}{2}\frac{\partial^2 w}{\partial k^2}}$$

  (Thanks to John-Joe Hosking of Imperial College)

- **Page 113** There should be an extra ΔK so that the formula reads

$$2\,N\,(d_2)|_{S=K}\,\Delta K$$

  (Thanks to Craig Nelson)

- **Page 124** should have $MinCoupon = -12\%$ not $MinCoupon = -1\%$.
  (Thanks to Jining Han)

- **Page 139** The definition of d_2 should have a minus sign:

$$z(k) = d_2 = -\frac{k}{\sigma\sqrt{T}} - \frac{\sigma\sqrt{T}}{2}$$

  (Thanks to James LaDue of Imagine Software)

- **Page 141** The upper limit of the $y-$integrations should be ∞ not T.
  (Thanks to Fahmi Zaidi)

- **Page 160** The VXB formula should read

$$\mathbb{E}_{T_1}[\sqrt{\langle x\rangle_{T_1,T_2}}] =: Y$$

  (Thanks to Jining Han)

- **Page 177** Add an entry: "Stochastic differential equation (SDE), 4,7,26,43,44,52,57,87,91."