

Final Take Home examination.

Due before end of term.

Q1. From a normal population with unknown mean and variance a sample of size 17 is drawn and the sample mean is found to be 0.13 while the sample standard deviation is 0.25. Are the observations consistent with the hypothesis that the population mean is 0 against the alternative that it is not? Use a 5% level of significance.

Q2. According to a model, the probability of of a single individual being of type  $AA$ ,  $Aa$  or  $aa$  are given by  $p^2$ ,  $2p(1 - p)$  and  $(1 - p)^2$  respectively where  $0 < p < 1$  is an unknown parameter. In a sample of size 100, the frequencies were found to be 30, 40 and 30 respectively. Is the data consistent with the model at 5% level of significance?

Q3. For the family of distributions with densities on the interval  $[0, 1]$  given by

$$f(x, \theta) = \theta x^{\theta-1}$$

where  $\theta > 0$  is a parameter, calculate the Cramér-Rao lower bound. What is the maximum likelihood estimator of  $\theta$ ? Can you construct a consistent estimator using the mean? What is its asymptotic variance? If from a sample of size 50 the sample mean is calculated to be 0.39 construct a confidence interval for  $\theta$  with a 95% confidence level.

Q4. If a sample of 15 observations from some distribution was

2.3, 1.2, 1.8, 1.6, 1.1, 3.3, 2.5, 2.2, 1.9, 1.4, 0.8, 3.0, 2.1, 1.5, 0.5

is it consistent with the median of the distribution being 2?

Q5. An examination of daily returns over a period of 100 days yielded a sample correlation of 0.15 between two broad based indices of the Paris and New York markets. Is it consistent with the hypothesis that the two markets are uncorrelated? Is the test that you used robust enough for this situation?