Homework 7. Due April 7, 2005

Q1. $x_1, x_2, \ldots, x_n$ are n independent observations from a Normal Distribution with both mean and variance equal to an unknown parameter $\theta > 0$, i.e the probability density of a single observation is

$$f(x, \theta) = \frac{1}{\sqrt{2\pi\theta}} e^{-\frac{(x-\theta)^2}{2\theta}}$$

What is the maximum likelihood estimator $\hat{\theta}(x_1, \ldots, x_n)$ of θ?

Q2. A single observation X can be of type a, b or c with probabilities $\frac{\theta}{3}$, $\frac{2\theta}{3}$ and $1 - \theta$ respectively. Here θ is an unknown parameter with $0 \leq \theta \leq 1$. Ten independent observations resulted in 4 of type a and 3 each of types b and c. What is the maximum likelihood estimator of θ?