Probability and Statistics

Home Work due March 3, 2005.

Q1. If X is a random variable having a normal distribution with mean 0 and variance 1, i.e. having the density

$$f(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}$$

show that the probability density of the random variable $Y = X^2$ is given by

$$f_1(x) = c_1 e^{-\frac{x}{2}} x^{\frac{1}{2}-1}$$

and c_1 is a normalization constant so chosen that $\int_0^\infty f_1(x)dx = 1$.

Q2. Show that the probability distribution of $S = X_1^2 + \cdots + X_n^2$ is given by

$$f_n(x) = c_n e^{-\frac{x}{2}} x^{\frac{n}{2}-1}$$

by identifying the moment generating functions

$$\int_0^\infty e^{\theta x} f_n(x)dx = \frac{1}{(1-2\theta)^{\frac{n}{2}}} = \Big[\int f_1(x)e^{\theta x}dx\Big]^n$$

Q3. If S is the number of heads in 100 independent tosses of a fair coin, use Tchebychev's inequality to estimate how small

$$P[S \geq 75]$$

is.