Real Variables Fall 2007.

Assignment 10. Due Nov 12.

Problem 1. For any set subset $A \subset X$ in a metric space we define

$$d(x, A) = \inf_{y \in A} d(x, y)$$

Show that $d(x, A) = 0$ if and only if $x \in \overline{A}$. In particular show that, if A is closed then $x \in A$ if and only if $d(x, A) = 0$.

Problem 2. If (X, d) is a complete metric space then show that (A, d), where $A \subset X$ is a subset, is complete if and only if A is closed. On the other hand if $G \subset X$ is an open subset of a complete metric space (X, d), then show that

$$D(x, y) = d(x, y) + \left| \frac{1}{d(x, G^c)} - \frac{1}{d(y, G^c)} \right|$$

is an equivalent metric on G and that (G, D) is complete. If we replace G by a set $A = \cap G_n$, a countable intersection of open sets, show that the original metric d can again be modified on A so that A is complete under the new metric.