Homework Set 9. Due April 19, 2004.

1. Suppose $\pi_h(x, dy)$ is a Markov Chain transition probability on R such that

$$\lim_{h \to 0} \sup_x \frac{1}{h} \int (y-x)\pi_h(x, dy) - b(x)| = 0$$

$$\lim_{h \to 0} \sup_x \frac{1}{h} \int (y-x)^2\pi_h(x, dy) - a(x)| = 0$$

$$\lim_{h \to 0} \sup_x \frac{1}{h} \int |y-x|^3\pi_h(x, dy)| = 0$$

for some smooth functions $a(x)$, $b(x)$ and $u(t, x)$ is a smooth solution of

$$u_t = \frac{1}{2}a(x)u_{xx} + b(x)u_x, u(0, x) = f(x)$$

Then, show that

$$\lim_{\substack{n \to \infty \\ h \to 0 \\ nh \to t}} \int f(y)\pi_h^n(x, dy) = u(t, x)$$

2. If $u(t, x)$ is a smooth solution of

$$u_t = \frac{1}{2}u_x x + b(x)u_x, u(0, x) = f(x)$$

and v is a smooth solution of

$$v_t = \frac{1}{2}v_{xx} + (b(x) + c(x))v_x, v(0, x) = f(x)$$

show that

$$\sup_x |u(t, x) - v(t, x)| \le t \sup_x |c(x)| \sup_{t,x} |u_x(t, x)|$$