

**Homework Set 4. Due March 8.**

**1.** Verify that the function

$$u(t, x) = \frac{x}{t^{\frac{3}{2}}} e^{-\frac{x^2}{2t}}$$

satisfies the heat equation

$$u_t = \frac{1}{2} u_{xx}$$

with the initial condition  $u(t, x) \rightarrow 0$  as  $t \rightarrow 0$  for all  $x$ . Does this contradict uniqueness? How do you reconcile this?

**2.** Show that the solution to the heat equation  $u_t = \frac{1}{2} u_{xx}$  with initial condition  $u(0, x) = p(x)$  where  $p$  is a polynomial is given by a polynomial in  $t$  and  $x$ . Do explicit calculations for  $p(x) = x, x^2, x^3$  and  $x^4$ .