

Homework Set 4. Due March 8.

1. Verify that the function

$$u(t, x) = \frac{x}{t^{\frac{3}{2}}} e^{-\frac{x^2}{2t}}$$

satisfies the heat equation

$$u_t = \frac{1}{2} u_{xx}$$

with the initial condition $u(t, x) \rightarrow 0$ as $t \rightarrow 0$ for all x . Does this contradict uniqueness? How do you reconcile this?

2. Show that the solution to the heat equation $u_t = \frac{1}{2} u_{xx}$ with initial condition $u(0, x) = p(x)$ where p is a polynomial is given by a polynomial in t and x . Do explicit calculations for $p(x) = x, x^2, x^3$ and x^4 .