

## Homework Set 9

Due Nov 24

Q1.  $V$  is a real vector space. Show that the intersection  $C = \bigcap_{\alpha} C_{\alpha}$  of an arbitrary family of closed convex sets  $\{C_{\alpha}\}$  in  $V$ , is again a closed convex set in  $V$ . Show that any closed bounded ( compact) convex set is an intersection of half spaces. A half space is a closed convex set of the form

$$\{x : l(x) \leq a\}$$

where  $l$  is a linear function on  $V$ .

Q2. Given a closed convex set  $C$ , a tangent plane at  $x \in C$  is a hyperplane  $\{y : l(y) = a\}$ , such that  $l(x) = a$  and either  $l(y) \leq a$  for all  $y \in C$  or  $l(y) \geq a$  for all  $y \in C$ . Given a closed convex set  $C$ , and a linear functional  $l$  show that there is at least one point  $x \in C$  and a real number  $a$  such that  $\{y : l(y) = a\}$  is a tangent plane. Show that at every boundary point  $x \in \delta C$  there is at least one tangent plane.