Homework Set 7

Due Nov 10

Q1. Consider the matrix

$$\mathbf{A} = \begin{pmatrix} < x_1, x_1 > & < x_1, x_2 > & \cdots & < x_1, x_n > \\ < x_2, x_1 > & < x_2, x_2 > & \cdots & < x_2, x_n > \\ \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots \\ < x_n, x_1 > & < x_n, x_2 > & \cdots & < x_n, x_n > \end{pmatrix}$$

where $x_1, x_2, \ldots, x_n$ are n vectors in a complex inner product space. Show that $\mathbf{A}$ is Hermitian and positive semidefinite. It is positive definite if and only if the n vectors $x_1, x_2, \ldots, x_n$ are linearly independent.

Q2. $\mathbf{A}$ is a Hermitian and positive definite linear transformation of a complex vector space $V \to V$. Show that $\langle\langle x, y \rangle\rangle = < x, \mathbf{A}y >$ defines a new inner product on V. Can you express the new adjoint $\widehat{\mathbf{B}}$ of a transformation $\mathbf{B}$ in terms of the old adjoint $\mathbf{B}^*$ and $\mathbf{A}$. When is $\mathbf{B}$ self adjoint with respect to the new inner product?