Homework. Set 6

Due Nov 3, 2004.

Q1. If A and B are two linear transformations of $V \to V$ that commute i.e. $AB = BA$, then show that the eigen space

$$W = \{v : Av = \lambda v\}$$

of A corresponding to any scalar λ, is an invariant subspace of B. In other words

$$BW \subset W$$

Is the same true of the generalized eigen spaces

$$W^k = \{v : (A - \lambda I)^k v = 0\}$$

for $k \geq 2$?

Q2. A linear transformation A of $V \to V$ is called semi-simple if for every subspace $W \subset V$ which is invariant, i.e with the property $AW \subset W$ there is a complementary invariant subspace W'. Show that every semi-simple linear transformation over the complex numbers can be represented by a diagonal matrix in some basis. Conversely any transformation that is diagonal in some basis is semi-simple.