Homework Set 5. due Oct 20.

Q1. A subspace $W \subset V$ is called an invariant subspace of a linear transformation $A : V \to V$ if $Ax \in W$ when ever $x \in W$, i.e A maps W into itself. Show that any linear transformation A on a vector space V of dimension n over the real numbers has an invariant subspace of dimension 1 or 2. If n is odd, then it has at least one 1 dimensional invariant subspace, i.e an eigen-vector.

Q2. Let V be a vector space of dimension n and V' its dual. Let $\{e_i : 1 \le i \le n\}$ be a basis of V'. For $i < j$, $f_{i,j} = e_i \wedge e_j$ are viewed as antisymmetric bilinear functionals

$$f_{i,j}(v_1, v_2) =< v_1, e_i >< v_2, e_j > - < v_1, e_j >< v_2, e_i >$$

on $V \times V$. Show that $f_{i,j}$ are linearly independent and constitute a basis for the vector space of antisymmetric bilinear functionals on $V \times V$.