Homework Set 3. Due Oct 6, 2004

Q1. What is the rank of the matrix

$$\begin{pmatrix} 1 & x_1 & x_1^2 \cdots & x_1^{n-1} \\ 1 & x_2 & x_2^2 \cdots & x_2^{n-1} \\ \cdots & \cdots & \cdots & \cdots \\ 1 & x_n & x_1^2 \cdots & x_n^{n-1} \end{pmatrix}$$

Q2. If S and T are linear transformations of a finite dimensional vector space into itself and $V = ST$ is one -to-one and onto show that the same is true of S and T.

Q3. If S and T are linear transformations of a finite dimensional vector space into itself . Give upper and lower bounds for the rank $r(V)$ of $V = ST$ in terms of the ranks $r(S)$, $r(T)$ of S and T respectively. Give examples to show that the bounds are the best possible.