Homework Set 2. Due Sep 29, 2004

Q1. A linear map from $V \to V$ is called a projection if $P^2 = P$. Show that if N is the null space of P, i.e $N = \{x : Px = 0\}$ and R is the range of P, i.e. $R = \{x : x = Py, y \in V\}$, then $V = N \oplus R$ or V is a direct sum of N and R.

Q2. If V is the space of all continuous functions $f(x)$ on the closed interval $[0, 1]$ and $W \subset V$ consists of those continuous functions f that satisfy $f(0) = 0$, what is the dimension of the quotient space V/W. Can you find a complementary subspace X such that $V = W \oplus X$. Can you find all possible such choices of X?