If we have two nonnegative functions $f(x)$ and $g(x)$ and we compare their behavior as $x \to \infty$, i.e for large values of x, there are many possibilities. $f(x)$ can get much larger than $g(x)$ which means

$$\lim_{x\to\infty} \frac{g(x)}{f(x)} = 0$$

or the other way around $g(x)$ gets much bigger than $f(x)$ or

$$\lim_{x\to\infty} \frac{f(x)}{g(x)} = 0$$

or they may be of the same size, i.e the ratios are neither too large nor too small which is the same as saying $\frac{f(x)}{g(x)}$ and $\frac{g(x)}{f(x)}$ remain bounded.

The behavior can be indefinite. For example comparing $f(x) = e^{x \sin x}$ to $g(x) = 1$. When $\sin x > 0$ $f(x)$ is a lot bigger than 1, but when $\sin x < 0$ it is a lot smaller. Since $\sin x$ oscillates between ± 1, the situation keeps changing for ever,

One can consider similar behavior when $x \to 0$ or any other value.

Example: $f(x) = 1 + x^2$, $g(x) = 2 + x$. As $x \to \infty$, $f(x)$ is a lot bigger, But $x \to 0$, the ratio is $\frac{1}{2}$ or $\frac{2}{1}$. But as $x \to -2$, $g(x)$ is a lot smaller than $f(x)$.

Example: $f(x) = 1 - \cos x$, $g(x) = x$. By L'Hospital's rule, as $x \to 0$ f is a lot smaller. As $x \to \infty$ f is at most 2 and so is a lot smaller than x.

Example: $f(x) = 1 - \cos x$, $g(x) = x^2$. By L'Hospital'sExample: $f(x) = 1 - \cos x$, $g(x) = x$ rule now $\frac{f(x)}{g(x)} \to \frac{1}{2}$ so they are of the same size.

If f is a lot bigger we write $f(x) >> g(x)$ or $g(x) << f(x)$.

The Quiz on monday will be of the following type $f(x)$ and $g(x)$ will be given. You have to decide as $x \to \infty$ or some other value specified in the question if $f \simeq g$, or $f >> g$, or $f << g$ or none of the above. None of the above means the behavior keeps changing.

Example: $f(x) = xe^{2x}$, $g(x) = (x^2 + 1)e^x$. As $x \to \infty$ and as $x \to 0$.

Ans: As $x \to 0$ $f << g$ (because $f(0) = 0$ and $g(0) = 1$. As $x \to \infty$, $f >> g$, because

$$\frac{g(x)}{f(x)} = \frac{x^2+1}{x} e^{-x} = (x + \frac{1}{x}) e^{-x} \to 0$$