The Mathematics of Modern Physics

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Prerequisites: Calculus I, II and III; some knowledge of Linear Algebra is also desirable.

Description

Mathematics and Physics have a long history of fruitful collaboration: Much of modern mathematics originated in response to the needs of emerging physical theories; and large areas of modern physics, such as General Relativity and Quantum Mechanics, are fundamentally mathematical in nature. This course will explore this long–dating relationship, using the evolution of modern physics as a guide for a tour through a number of mathematical topics. The purpose of the course is twofold: by studying the mathematical structure underlying modern physics, it is expected that the students will develop an appreciation for the subtlety of physical thinking, while enriching their knowledge of mathematics and of the way in which it can help us improve our understanding of the world.

We shall start with Newton's mechanics, which is intimately connected to the origin of calculus and ODE's. We'll study the variational (Lagrangian) and simplectic (Hamiltonian) formulations, and their beautiful mathematical structures. Later in the course, we'll learn that these formulations enable us to make analogies between mechanics and wave propagation, that are at the very heart of Quantum Mechanics and General Relativity. Then we shall consider Electromagnetism, learning how the requirement of mathematical consistency between various experimental laws, written as PDE's, led Maxwell to give the so far artificial idea of a field a new real dimension. Next we shall study Special Relativity, helped by some Linear Algebra, and General Relativity and its very geometrical view of the world. Finally, we shall look at the mathematical structure of Quantum Mechanics, including Fourier analysis, dispersive waves, linear operators, complex inner products, and eigenvalue problems.

By the last few weeks of the course, each student will have chosen a topic for more intensive study, and will lecture on it to the class. The final grade will be based on this lecture, on a written report about its content, an on two or so homework assignments given throughout the semester.

References

There is no basic text for the course. References and notes will be provided in class. However, the instructor strongly recommends that all students with an interest in Physics adquire The Feynman Lectures on Physics, a book that will provide delightful and instructive reading not only for this course, but also for many years to come. Another very good elementary physical reference, with an emphasis on relativity, is Einstein and Infeld's The evolution of Physics. A more advanced, yet very readable book, that we'll use much in the course, is O. Buhler's A brief introduction to classical, statistical and quantum mechanics.

On the mathematical side, two excellent and widely different books on Linear Algebra are P. Lax's and G. Strang's. A very good but a bit too advanced book on the Hamiltonian and other related formalisms is V. Arnold's Mathematical Methods of Classical Mechanics. Another very good advanced text that will provide some important mathematical tools for Quantum Mechanics is Bender and Orszag's Mathematical Methods for Engineers and Scientists. Any elementary book on PDE's, such as Habermann's Elementary Applied Partial Differential Equations, will provide additional material on the wave equation, potential theory and Sturm-Liouville problems.