PULSATILE FLOW ACROSS THE MITRAL VALVE: HYDRAULIC, ELECTRONIC AND DIGITAL COMPUTER SIMULATION

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Abstract: Classical hydraulics, electronic analogs, animal studies, and numerical solutions to the equations of motion using a digital computer, have been applied to the investigation of atrioventricular flow dynamics. The results are complementary, consistent, and have served to elucidate the patterns of flow and the overall system dynamics. The role of inertia is shown to be significant in understanding instantaneous pressure-flow dynamics and cusp motion; and the quasi-steady nature of the time average properties have been investigated.

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INTRODUCTION

The mitral valve is situated between the two chambers of the left heart and permits unidirectional flow from atrium to ventricle. In the normal individual, events associated with the closure of the mitral valve give rise to the first heart sound, and valvular diseases lead to variations in the quality, intensity, timing and duration of sounds associated with mitral flow. Diseases of the mitral valve also lead to a disturbed relationship between pressure and flow. The elevated left atrial pressure associated with mitral stenosis especially at high flow rates is both an important diagnostic tool and also a direct cause of the patient's distress during exercise. The interpretation of heart sounds and pressure waveforms represents an important aspect of cardiological diagnosis, but this interpretation can only proceed on a rational basis when the relationship between these data and the flow dynamics is clearly understood. Since instantaneous flow cannot ordinarily be measured in man, we must turn to a variety of other systems to gain such understanding. In this paper we attempt to present a comprehensive analysis of the dynamics of flow across the mitral valve. Physical modeling, mathematical modeling, and animal experiments are all used to build up a consistent picture of the mechanisms that govern the behavior of the mitral valve in its normal and pathological states.

HYDRAULIC ANALOG

The anatomy of the left heart is shown in Fig. 1, and indicates that the mitral valve and its ring represents an area reduction between the atrium and ventricle, which may be likened to a nozzle or an orifice. As the chambers change size during a cycle, the size of the opening changes and the respective area ratios also change, but essentially, the geometry is still similar to an area reduction in a pipe. One is thus led to consider the equation governing flow across an orifice as a first approximation to the physiological system:

$$\Delta p = kQ^2 , \qquad \text{or} \qquad Q = CA\sqrt{\Delta p} \qquad (1)$$

This hydraulic equation was applied with some success by Gorlin and Gorlin (1)[1] in 1951 to the problem of estimating the area of a stenotic heart valve. Roderigo and Snellin (2) presented a firmer theoretical base and also substantiated the results shortly thereafter; this method is now used routinely in cardiological diagnosis.

There are several obvious objections to this approach: 1. Equation (1) is derived from the steady flow Bernoulli equation and neglects the influence of local acceleration. This assumption would appear to be questionable in light of the large amplitude pulsations found within the heart. 2. The area, A, of the normal orifice changes with time within the heart. 3. The discharge coefficient, C, is a weak function of the Reynolds number and it, too, changes with time. 4. Finally, we do not know if the flow is quasi-steady in relation to the energy losses. That is, it is not known if the discharge coefficient at each instant of time in an unsteady flow is the same as it would be in a steady flow with an equivalent Reynolds number. We therefore examined a pressure-flow relation which included an inertial term:

$$\Delta p = AQ|Q| + BdQ/dt \qquad (2)$$

where A and B are resistive and inertial coefficients respectively and the flow is multiplied by its absolute value rather than squared as in equation (1) in order to maintain the proper phase between the dissipative aspects of the

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1 Numbers in parentheses designate References at the end of the paper.

pressure-flow relation.

Equation (2), or its equivalent, has been derived in a number of ways, all of which rely on momentum considerations, and it has been applied to the unsteady flow problem by many investigators. Burger et al. (3) were the first to attempt to model a stenosis by studying sinusoidal flow across an orifice and applying equation (2). They correctly measured a hysteresis loop in the pressure-flow graph (although they erred in the direction of the loop) and they attempted to explain this result on the basis of a time-dependence between turbulent losses and unsteady flow. This so-called "hysteresis loop" arises from the inertial component in equation (2): when the flow is increasing, the derivative is positive and when the flow is decreasing, it is negative. Since the resistive component of the pressure gradient in equation (2) is always in phase with the flow, it follows that the pressure gradient for an increasing positive flow must be greater than that for a decreasing positive flow; hence, an apparent "hysteresis loop".

Lahey and Shiralkar (4) investigated the application of equation (2) to an exponentially decaying transient flow across an orifice. They found that for conditions of low flow rate or very rapid transients the inertial correction becomes appreciable. Thus, while their results are not directly applicable to a pulsatile flow, they indicate the need to investigate further the problem of fluid inertance.

Perhaps the most comprehensive study of large amplitude, pulsatile, incompressible flow across an orifice was presented by Moseley (5). The main thrust of his study, and of most engineering studies in this area, is toward the response of orifice meters to pulsations: that is, toward determining the magnitude of the error in flow rate as inferred from the measured pressure gradient. The problem here is that measuring systems which give the time average of a variable are subject to the "square root error"; namely, the square root of the mean, is not equal to the mean of the square root. Moseley (5) generated sinusoidal flows with and without a mean level; assumed negligible temporal acceleration; found the square root error to correctly express the error within experimental uncertainty; and concluded that the flow must be quasi-steady.

Since we are interested primarily in the instantaneous relations between pressure and flow, we have studied the dynamics of orifice flow as a preliminary to the study of intracardiac events (6).

Experimental Methods: Orifices of 1/8, 1/4, 3/8 and 1/2 inch diameters were mounted in a 1 inch diameter tube and subjected to steady flow, simple sinusoidal flow, and sinusoidal flow with a mean level. The range of frequencies and flow amplitudes included any found in the normal, stressed, or pathologic in vivo system. The instantaneous pressure difference and flow across the orifice were measured with Statham strain gage transducers and with a Carolina Medical Electronics Electromagnetic Flowmeter.

Results: A typical result for a sinusoidal flow is shown in Fig. 2. The pressure difference which creates this flow is precisely that which would be predicted by equation (2):

$$\Delta p = C \sin\omega t |\sin\omega t| + D\cos\omega t \qquad (3)$$

when $$Q = \hat{Q}\sin\omega t$$

This relationship was confirmed for all flow rates, frequencies, and orifice sizes.

If equation (2) is integrated over any time interval such that $Q(t_1) = Q(t_2)$, the inertial term would make no contribution on that interval and the time average of the flow should be quasi-steady regarding the influence of the acceleration. If the discharge coefficient is also quasi-steady, then the time average of equation (1) taken between the times the flow crosses zero, should describe all the flow conditions across a given orifice. Such an approach is shown in Fig. 3 which correlates all the data for two orifices of equal diameter but different shape, on the basis of the square root of the mean pressure as a function of the mean flow. The data represent all frequencies, flow rates, and combinations of steady and pulsatile flow which include the physiological range of parameters. The correlation is excellent and indicative of all the other orifices tested. We conclude that a pulsatile flow is quasi-steady in regard to the discharge coefficient, and, when averaged over time, in regard to the local acceleration. It should be noted that in order to obtain the correct value for the discharge coefficient in equation (1), one must take the average of the flow squared and not the square of the average flow. For a given flow wave-form, however, the square root error will be constant and the conclusions derived from Fig. 3 are applicable. In summary, equation (2) adequately describes a pulsatile flow across a constriction including large amplitude pulsations.

ELECTRIC ANALOG

If equation (2) does describe the pressure-flow relations across an orifice, then it should be possible to model this hydraulic system with an electrical system. With voltage and current as the analogs of pressure and flow respectively, the wave-shapes and temporal relations between the parameters in both systems should be similar. An electric analog can then be used to test the validity of the concepts, and to predict the behavior of the hydraulic system. Such an analog is shown in Fig.4a, along with a definition of a square law resistor, Fig. 4b, which satisfies the non-linear dissipative properties of the system.

An operational amplifier circuit giving the voltage-current relations corresponding to the elements of Fig. 4a, is shown in Fig. 4c. The results corresponding to a simple sinusoidal flow are presented in Fig.5. The wave-forms and phase relations compare favorably with the results of the hydraulic analog as shown in Fig. 2. The analog yielded the same favorable results for all the conditions modelled.

THE PHYSIOLOGICAL SYSTEM

In order to investigate the pressure-flow dynamics across the natural mitral valve (7, 8), the left heart of large mongrel dogs was instrumented as shown in Fig. 6. An intracardiac electromagnetic flowprobe (Carolina Medical Electronics) was sutured to the mitral annulus and the wires brought out through the atrial appendage. Pressure was measured with Statham strain gage transducers connected to short stiff catheters as shown. In some dogs, catheter tip transducers (Konigsberg P-20) were used for pressure measurement and intracardiac phonocardiograms.

Fig. 7 is a reproduction of an oscillographic record and reveals the following points of interest: Inflow to the ventricle starts at the moment of pressure gradient reversal (LAP exceeding LVP). The gradient is small and rapidly goes to zero or slightly negative (LVP greater than LAP) thereby decelerating the flow so that it too approaches zero, but at a later time. An atrial contraction produces another favorable gradient which is followed by another period of forward flow thereby giving the mitral flow trace its bicuspid appearance. A strong ventricular contraction rapidly decelerates the flow and closes the valve sometime after the pressure difference has reversed.

The inertial character of this system is revealed by fact that flow follows pressure. The resistive character is indicated by the fact that the flow reaches its maximum after the gradient reaches zero rather than at that moment (as it would in a purely inertial system); and also by the fact that the forward gradient exceeds the reverse gradient so that not all the kinetic energy is recovered as pressure. These same characteristics were present at any heart rate and stroke volume. The physiological system thus seems to obey the relations described by equation (2).

ELECTRICAL ANALOG OF THE MITRAL VALVE

Fig. 8a presents an electrical analog of the pressure-flow relations for the natural mitral valve. During forward flow the valve is assumed to have the properties of an orifice, so that the pressure-flow relation is given by equation (2). Two paths are provided in parallel with the diode for backward flow. The first of these is a resistor-capacitor network representing the visco-elastic properties of the closed valve and the tissue that supports it. Current flowing into this network is stored in the capacitors, just as fluid may be stored in the ballooned valve leaflets. Such current (or the fluid it represents) is returned in the forward direction as the reverse voltage (pressure difference) dies away. Pathological backflow may also occur when the valve leaflets fail to close completely. Such backflow is governed again by equation (2) but with different constants. To allow for backflow we include a second path around the diode with dissipative and inductive (inertial) elements whose impedances add to those of the forward flow path to give the total impedance to backflow. An operational amplifier circuit to solve the equation relating voltage and current is shown in Fig. 8b. An oscillographic record of the driving pressure gradient which includes an "atrial contraction", and the resulting flow, is presented in Fig. 9. The similarities with the physiological results (Fig. 7) are strikingly clear. We have also varied the timing of "atrial and ventricular contractions" in the model and we find the same dynamic relations as in the physiologic system.

DISCUSSION

The inertial character of flow across the aortic valve has been known for more than a decade (9), but until recently, no one had considered the possibility that a similar situation prevailed for the mitral valve. With the pioneering work of Nolan et al. (10), and the corroboration of Folts et al. (11), Williams et al. (12) and ourselves (7, 8), we must now accept the fact that phasic flow across all heart valves will possess an inertial component. The full impact of this conclusion has yet to be felt by the medical community. For example, the first heart sound should occur several milliseconds after the A-V gradient reverses because flow lags pressure and the valve closes with flow and not pressure (13).

The approaches presented thus far in this report all have in common the fact that they describe the system in terms of lumped parameters. They are phenomenological approaches which reveal the overall system dynamics but not the details. In the concluding section we describe briefly a distributed parameter approach. The results of a numerical method for solving the equations of motion governing the fluid and cardiac tissue are presented below to complement the foregoing analyses.

DIGITAL COMPUTER SIMULATION

Our aim is to calculate the flow pattern of blood in the heart. To do this, we must take into account not only the forces which arise in the blood itself, but also the forces which arise in the muscular heart walls and the membraneous valve leaflets. The latter forces are determined by the configuration of the heart apparatus in space in a manner which is fixed for the valve and time-dependent for the active muscle.

The motion of the cardiac tissue is determined by the condition that it moves at the local fluid velocity. To a reasonable approximation, the density of the whole system is uniform, since the heart apparatus is nearly neutrally buoyant. Consequently the heart apparatus can be idealized as a specialized region of the fluid where extra forces are applied in addition to the usual fluid forces.

Based on such considerations, we have devised a computer method for constructing approximate solutions to the equations of motion of the blood-valve-heart system (14). We have applied this method to a two dimensional representation of a valve of the mitral type guarding the opening between muscular atrial and ventricular chambers. The flexible valve leaflets are restrained by chordae which connect the ventricular apex to the tips of the leaflets. We begin the calculation at the end of systole, with the atrium distended and the ventricle small. First, the ventricle relaxes; later, the atrium contracts; and finally the ventricle contracts. When the valve has closed we stop the calculation.

In Fig. 10 we show the results of a computer experiment in which the chordae are under tension throughout diastole. Fig. 10 a-d shows four frames all prior to atrial systole, while Fig. 10 e and f shows two frames during atrial systole. These six frames have been selected from a total of 16 and are not equally spaced in time.

Fig. 10a shows the configuration of the valve, atrium, and ventricle, at the beginning of this computer experiment. The chordae exist as a force between the ventricular apex and the cusp tip, and are not shown. In this frame the atrial and ventricular pressures are equal at the beginning of diastole. Subsequent relaxation of the ventricle allows the valve to open as in Fig. 10b. The inflow to the ventricle forms a jet which, at this stage, diffuses outward at the cusp margins and does not reach deep into the ventricle. A significant vena contracta is also evident, and a vortex can be seen forming about 2/3 of the way down each cusp on the atrial aspect. In Fig. 10c the vortex has moved to the tip of the valve and grown considerably in strength; its streamlines are beginning to move the valve toward closure. The jet extends between the vortices and reaches deeper into the ventricle. In Fig. 10d the inflow is slowing down and the vortices are much more prominent; their streamlines sweep the valve rapidly toward closure. Part of the jet persists between the vortices, but most of its streamlines no longer connect with the atrium. Instead they circulate around the vortices and participate in valve closure. This brings out the close relationship between the "broken jet" theory of Henderson and Johnson (15) and the vortex theory of Leonardo da Vince (16).

In atrial systole the same qualitative phenomena are repeated. In Fig. 10e we see the jet re-established by atrial systole, and it now reaches deeper into the ventricle than before. Between 10e and 10f the valve is re-opened to some extent, vortices are formed at the valve tips, and closure begins again. In 10f we see a very late stage of atrial systole, with the jet broken and the vortices moving the valve toward closure.

DISCUSSION

From a digital computer solution to the equations of motion we have predicted the flow patterns of the blood as it flows across the mitral valve without assuming a geometry for the flow path. This is a powerful method whose results are not only consistent with the findings of in vivo (17) and model studies (18, 19), but also elucidate the flow characteristics. As a consequence, we can identify the streamlines which circulate as a vortex and close the valve without reducing ventricular volume, and those which go from the ventricle to the atrium and reduce the volume, i.e., backflow. These motions could have been deduced from the lumped parameter approach: the inertial properties are noted from the pressure-flow relations, leading one to conclude that pressure must reverse before flow and thereby impose a force on the valve in a direction opposite to that of the flow; ergo, the valve should be closing before all the blood has entered the ventricle. The mathematical solution is an additional and welcome verification using a different approach.

The distributed parameter approach can also resolve apparent contradictions between different observations. For example: there would appear to be an inconsistency between the finding that the valve achieves most of its closure during diastole, and the finding that the flow is quasi-steady with a constant discharge coefficient. This question is readily resolved when we note the existence of the vena contracta (Fig. 10b), which produces an area reduction of approximately 60%. Since the contraction coefficient is the only significant component of the discharge coefficient, we may anticipate a valve "closure" to 60% of its fully opened area without any additional energy losses. In effect, a thin plate orifice has been converted to a nozzle of smaller diameter and similar discharge coefficient.

While the mathematical model is a powerful tool, it can never completely describe the physiological system in detail, and one must always approach the results with a healthy skepticism. A very useful method is to look at the computer results as a form of interrogation. For example: our numerical solution of the flow patterns without chordae show a valve which opens widely and does not start to close. That is, there is no strong interaction between the valve cusps and fluid. The inclusion of chordae under tension allows the vortex to develop and sweep the cusps toward closure. Bellhouse (18), on the other hand, has found in his model studies, that vortices will form and the valve move toward closure in the absence of tension on the chordae. It is not our intention at this time to reject one hypothesis in favor of the other, but rather, to suggest the need for definitive in vivo experiments in order to answer the question raised by the computer solution.

Finally, given the observation of the flow patterns shown in Fig. 10, we can postulate a role for atrial systole in valve closure: at slow heart rates, where flow ceases before atrial contraction, an atrial systole re-establishes a favorable gradient which imparts momentum to the blood in the direction opposite to the direction of valve closure, and the cusps seal without backflow.

SUMMARY

From the anatomy of the left heart and the physics of flow, we have developed a comprehensive approach to the investigation of atrioventricular flow dynamics. Classical hydraulics, electronic analogs, animal studies and digital computer methods have complemented each other and produced a series of consistent solutions. In addition, we may deduce from these solutions a role for anatomical features such as the chordae tendineae, which heretofore has not been clear. We have elucidated the patterms of flow and the overall system dynamics to show that temporal acceleration is highly significant in understanding the instantaneous pressure-flow relations and cusp motion. Because the flow is quasi-steady in respect to the time average properties and the discharge coefficient, the mean pressure and flow are related by the orifice meter equation.

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LEGENDS FOR FIGURES

Fig. 1. The atrioventricular configuration at the end of systole, just before the mitral valve opens; and at the end of diastole after the ventricle has filled. Note that the A-V valve always presents a reduction in area.

Fig. 2. Results of pressure difference (Δp) and flow (Q) across an orifice plate in a tube, when the flow is sinusoidal.

Fig. 3. The pressure-flow relations across two orifices of 3/8" diameter with shapes as shown. The data points include steady flow (SF), sinusoidal flow (PF) and sinusoidal flow with a mean level (PF c̄ SF). The regression equations are based on the time average of the flow and the square root of the time average of the pressure difference. See the text for further discussion.

Fig. 4a. The electric analog of an orifice; b. The definition of a square law resistor; and c. The operational amplifier circuit for solving the equations governing the pressure-flow relations across the analog.

Fig. 5. The pressure-flow relations across the electric analog of Fig. 4 when the flow is sinusoidal. Compare to Fig. 2, the hydraulic results.

Fig. 6. The left side of the canine heart instrumented to measure left ventricular pressure (LVP), left atrial pressure (LAP), mitral valve flow (MF) and aortic valve flow (A_oF). See the text for details.

Fig. 7. A recording of the atrioventricular pressure-flow relations in the dog heart. The two-peaked character of the mitral flow is due to an atrial systole in late diastole. The sharp point in the second and third mitral flow trace is an electrical artifact arising from atrial depolarization.

Fig. 8. a. An electrical analog of the mitral valve. In parallel with the diode are a storage path and a leakage path. b. The operational amplifier circuit which solves the equations governing the pressure-flow relations across the analog of a.

Fig. 9. The pressure-flow relations across the analog of Fig. 8 with infinite backflow impedance. The driving pressure gradient mimics the normal atrioventricular pressure difference of Fig. 7. Note the similarity of wave-forms and temporal relations between the analog and the in vivo results.

Fig.10. Digital computer solution of flow patterns across the mitral valve:

a. The configuration of the atrium, valve and ventricle at the beginning of diastole when the pressures in both chambers are equal.

b. The valve is fully open in early diastole, a vena contracta is evident, and a vortex is forming.

c. The vortex has moved to the valve tip and grown stronger so that its streamlines are moving the valve toward closure.

d. The jet has "broken" so that there is no flow out of the atrium and a large vortex is rapidly closing the valve.

e. An atrial systole re-establishes the jet and re-opens the valve.

f. As in d, the jet "breaks" again and the valve rapidly closes.

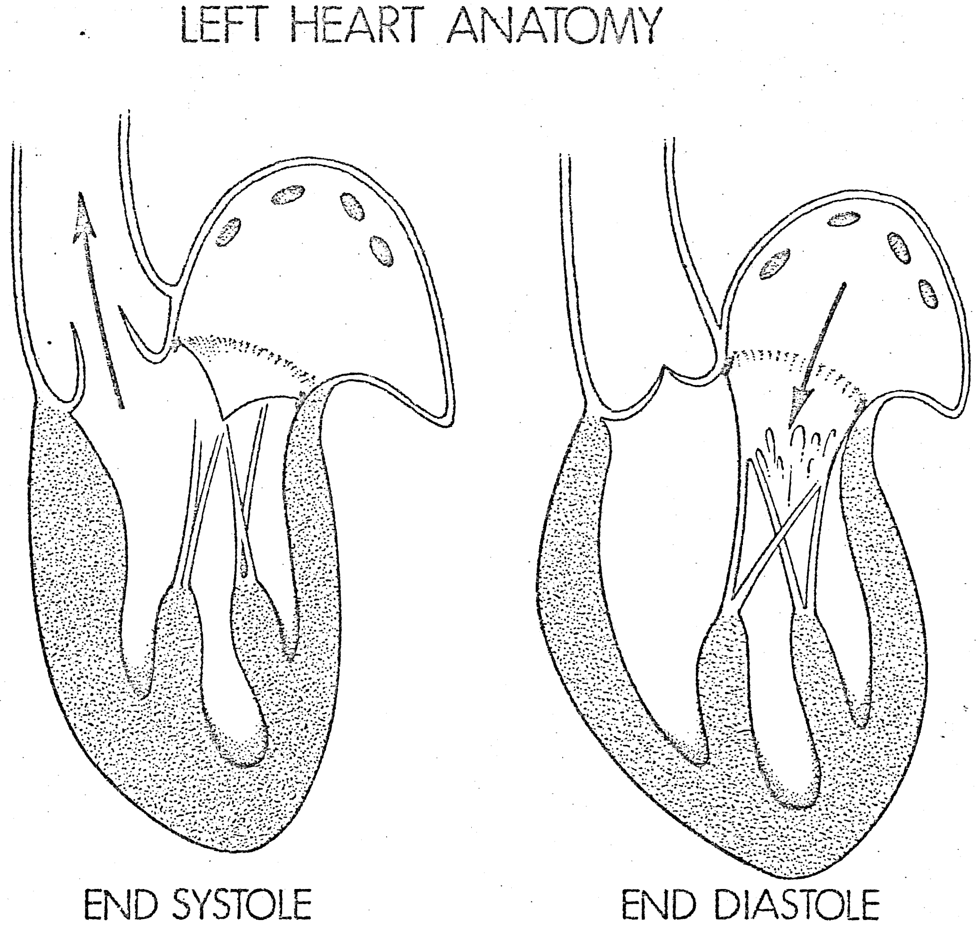


Figure 1

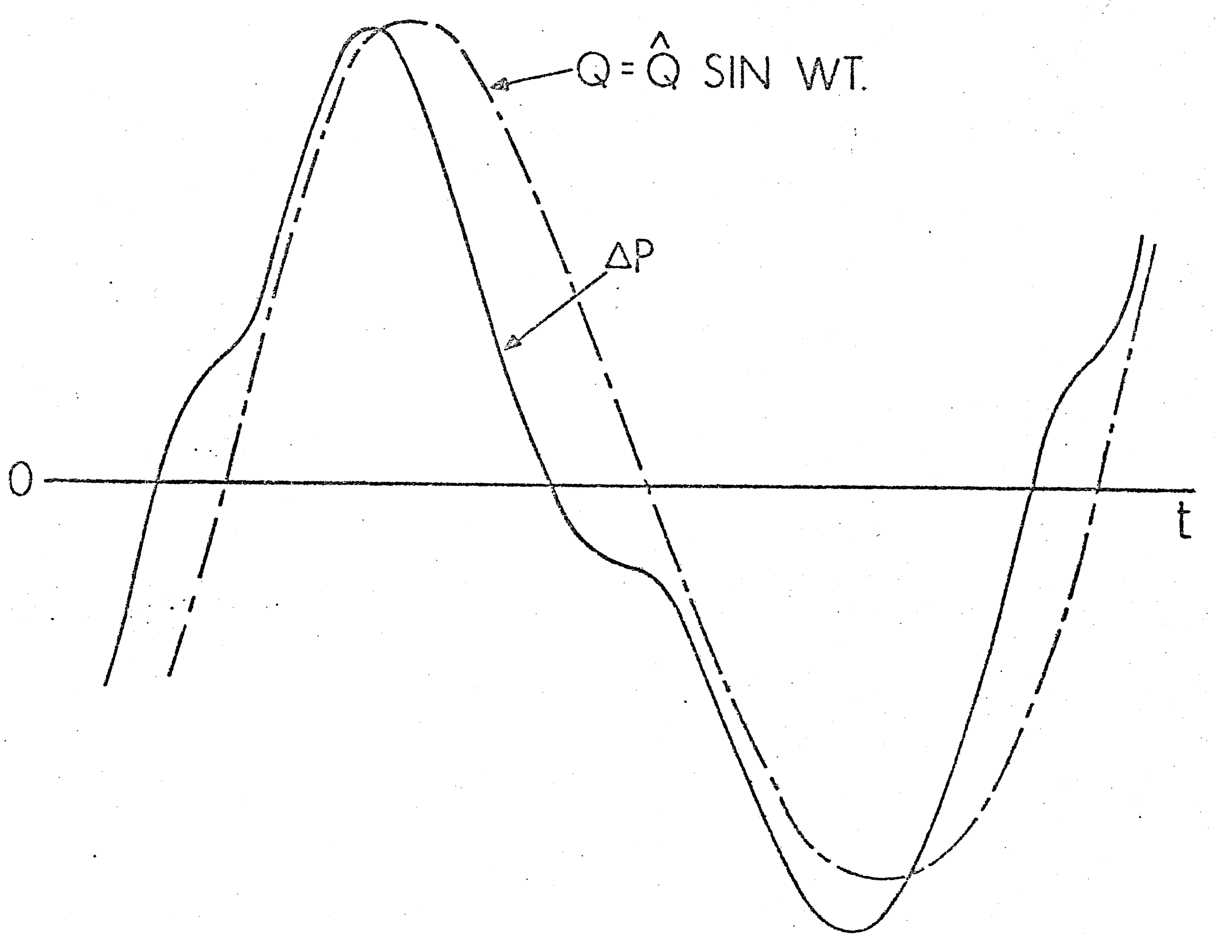
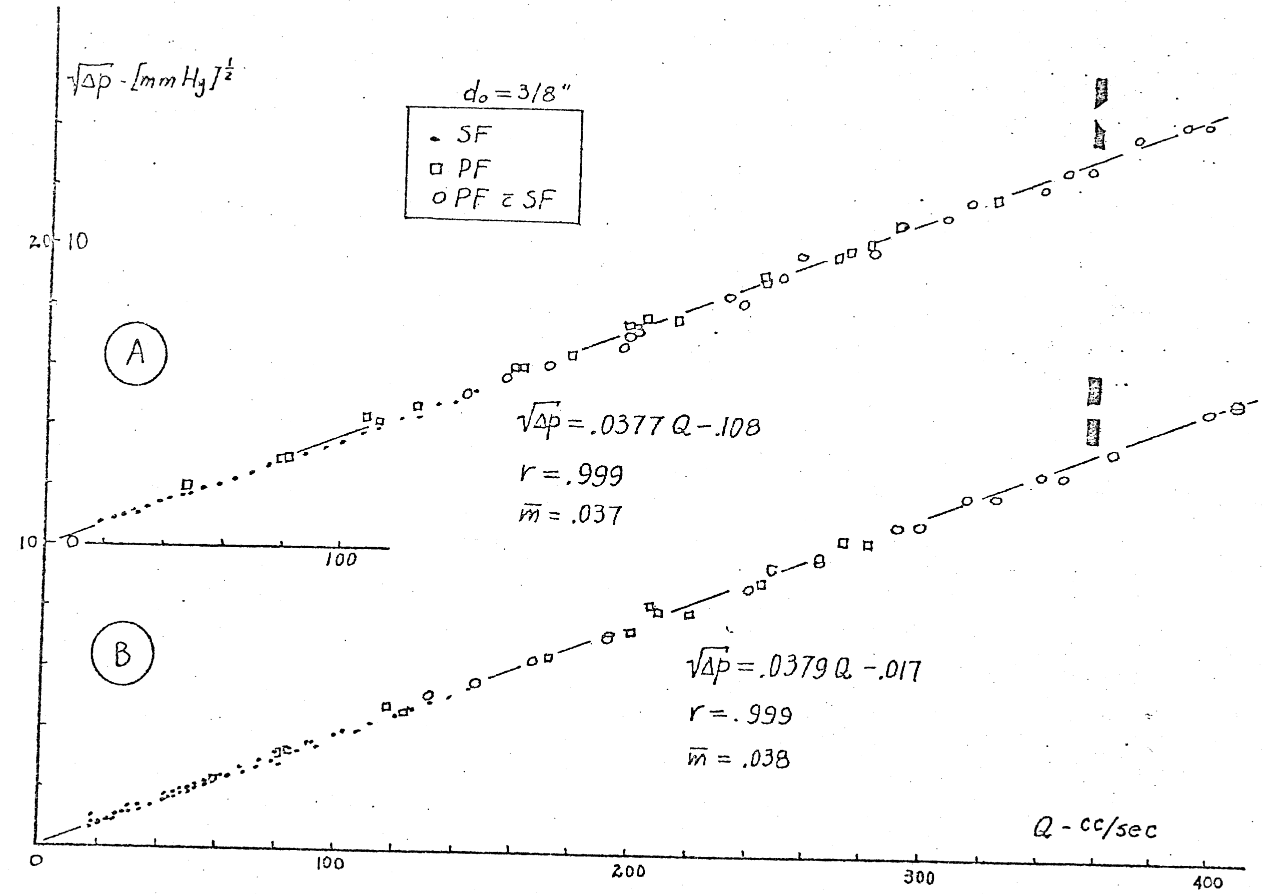


Figure 2



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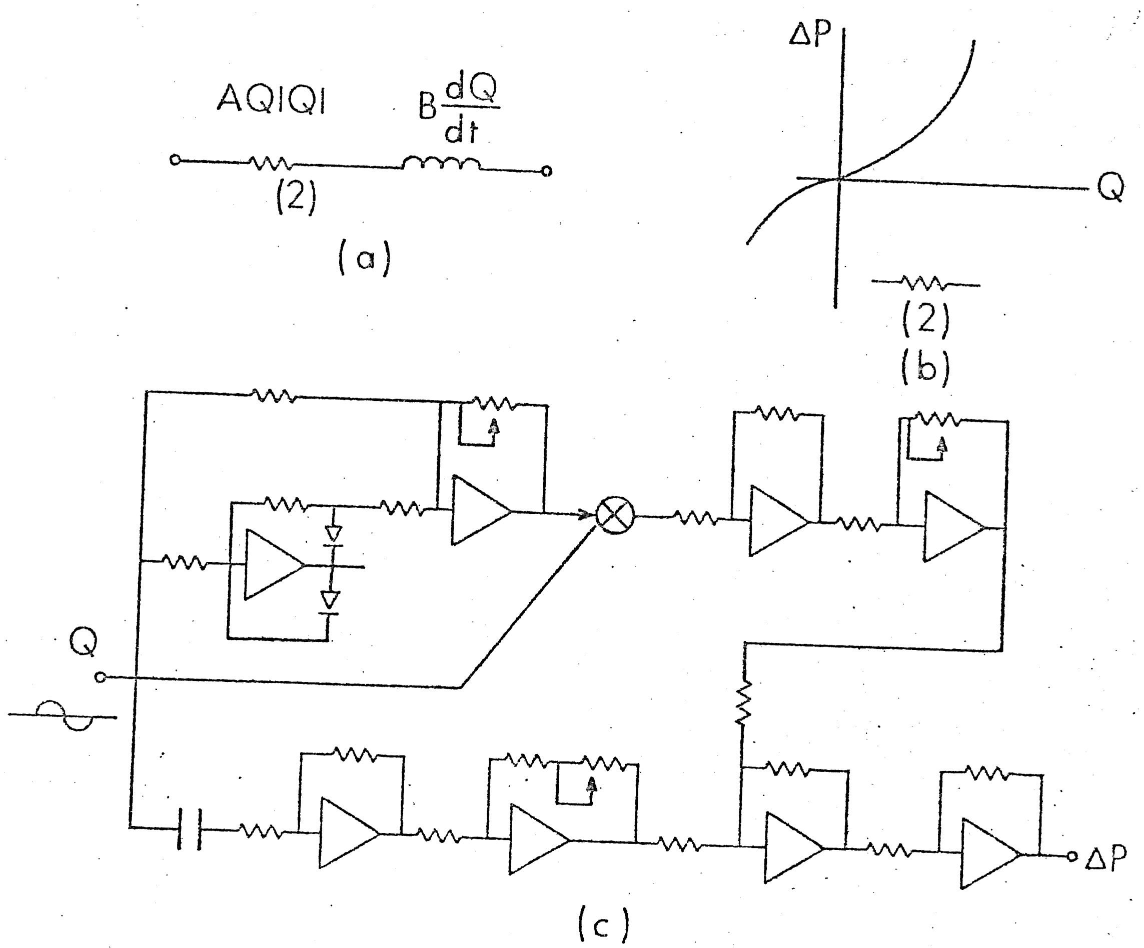

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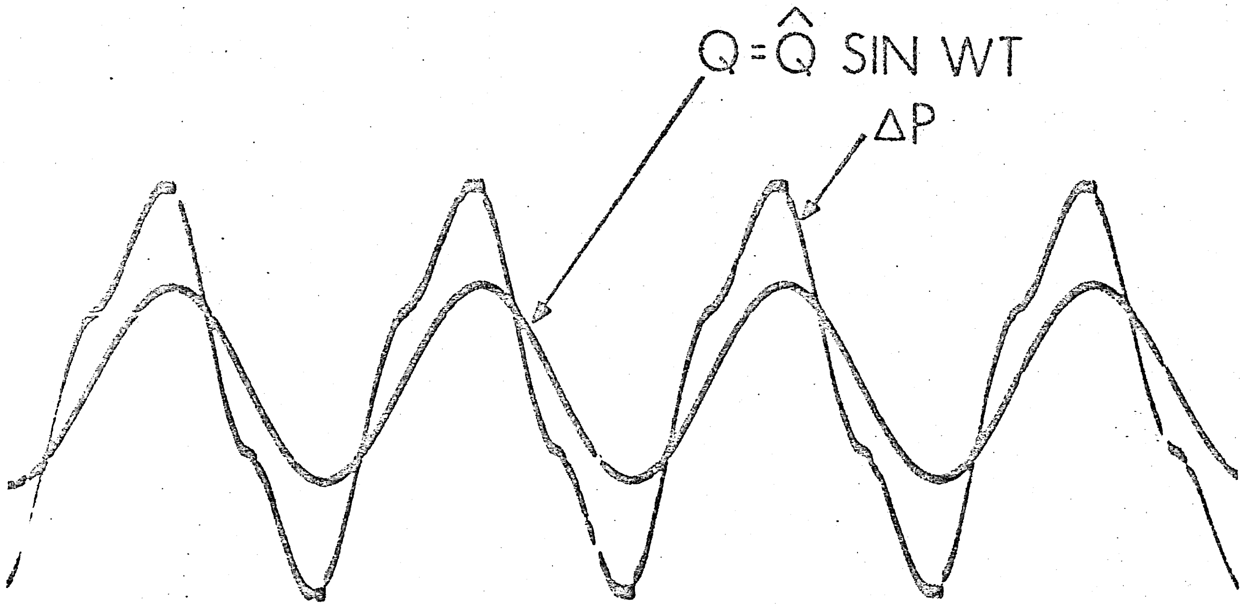

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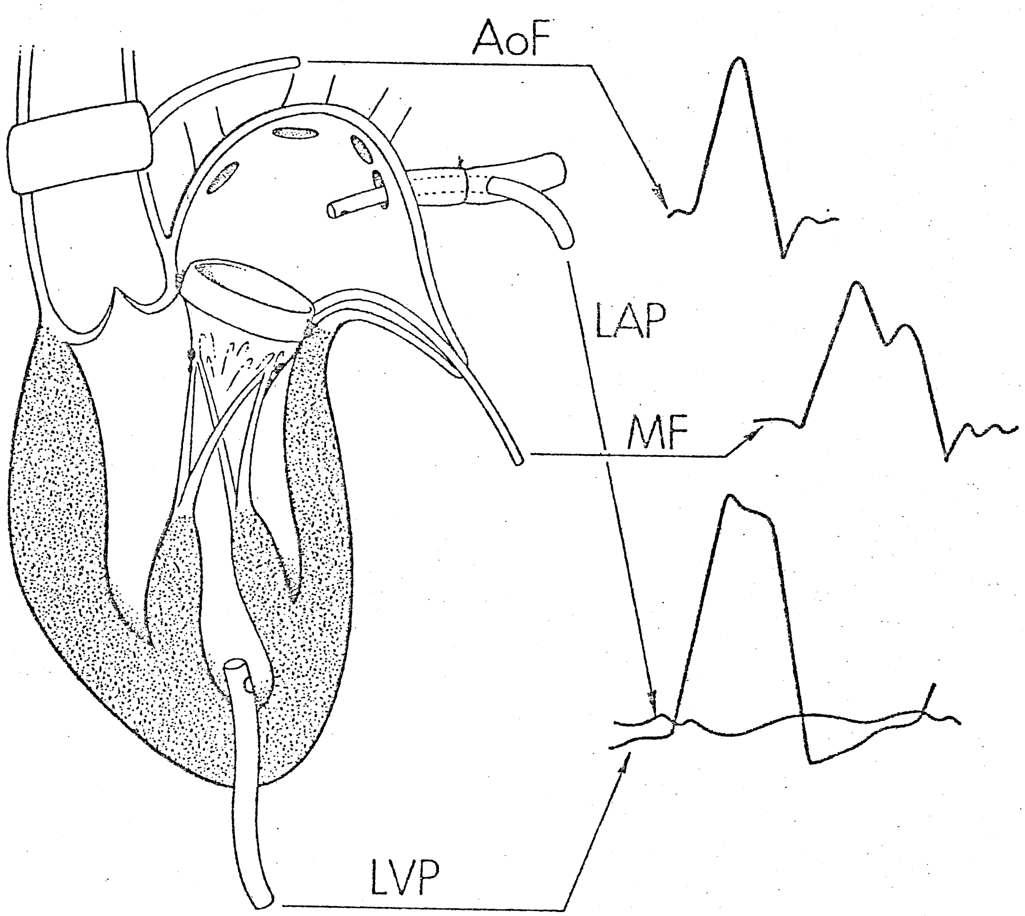

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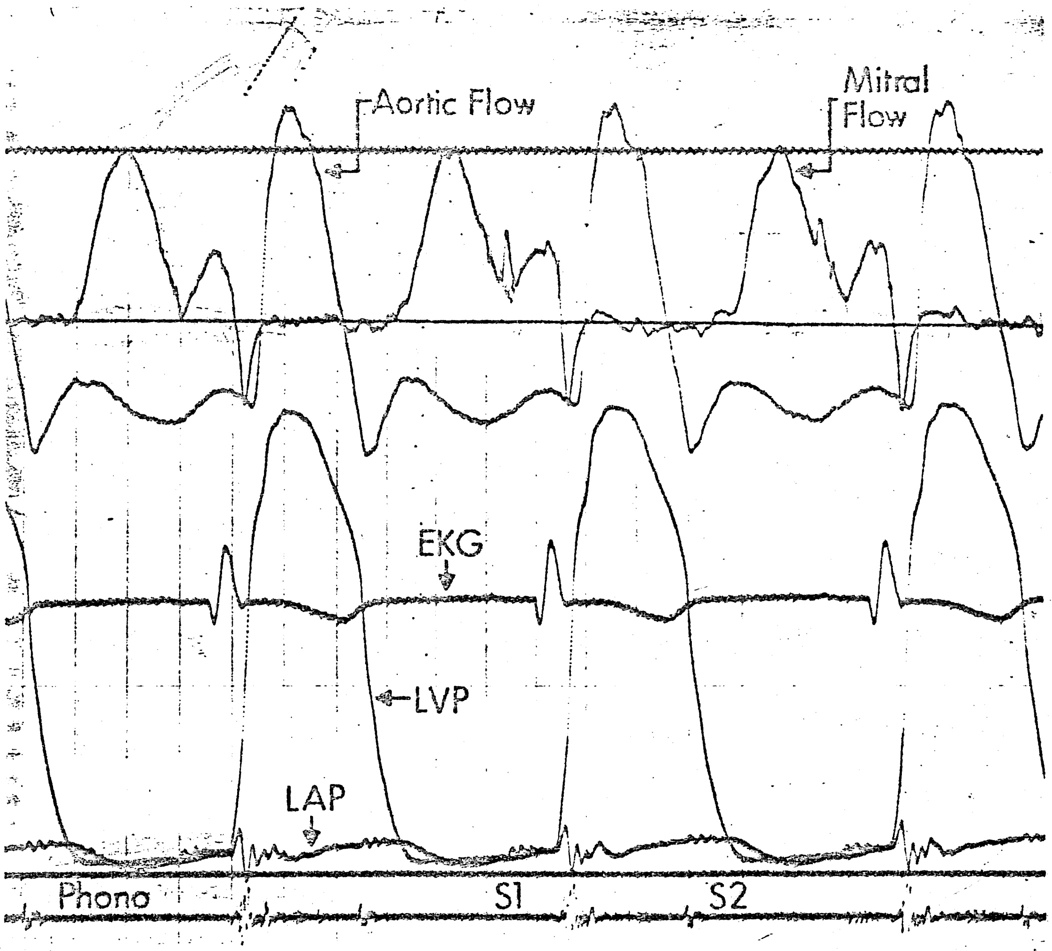


Figure 7

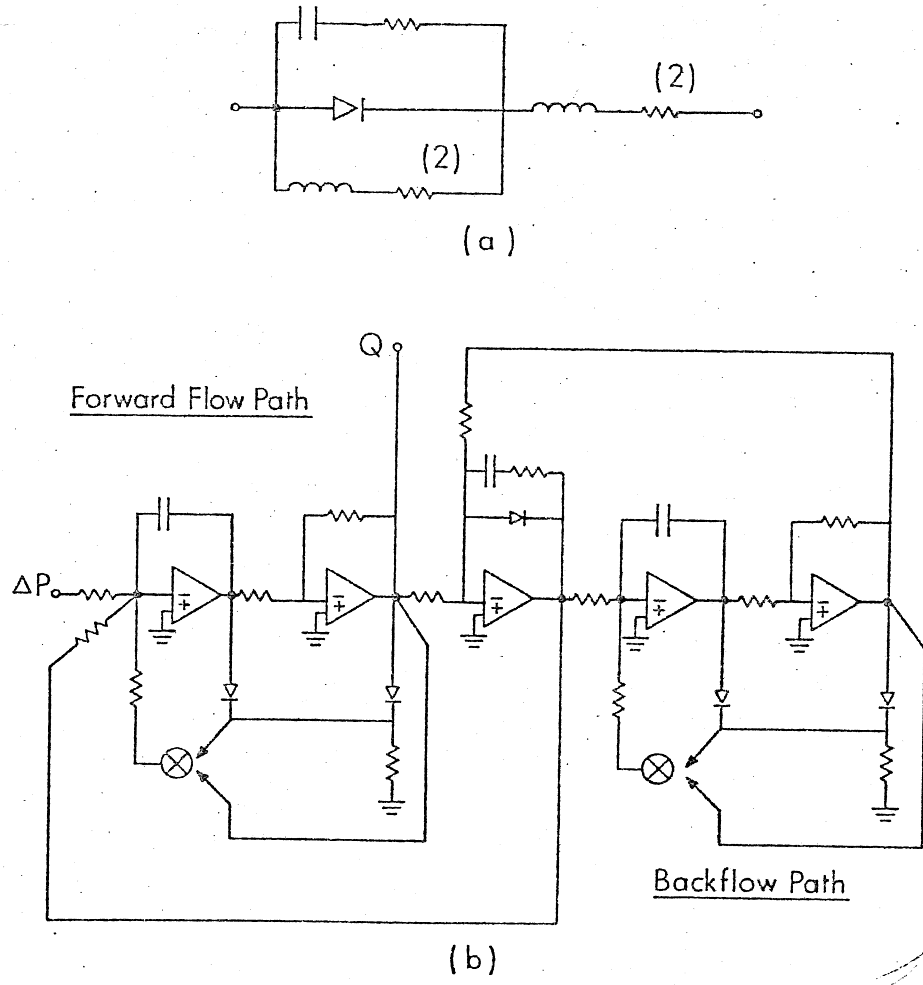


Figure 8

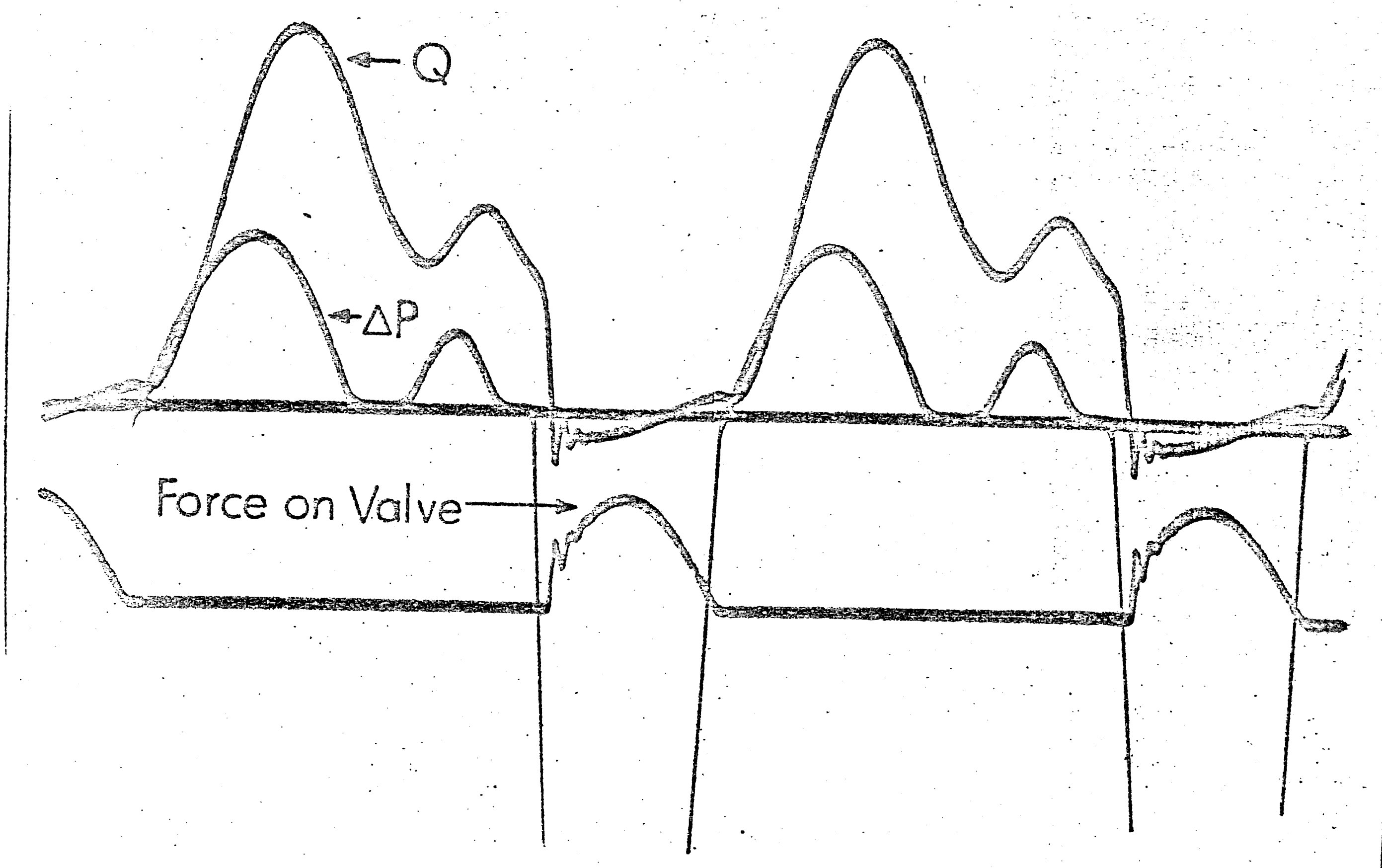


Figure 9

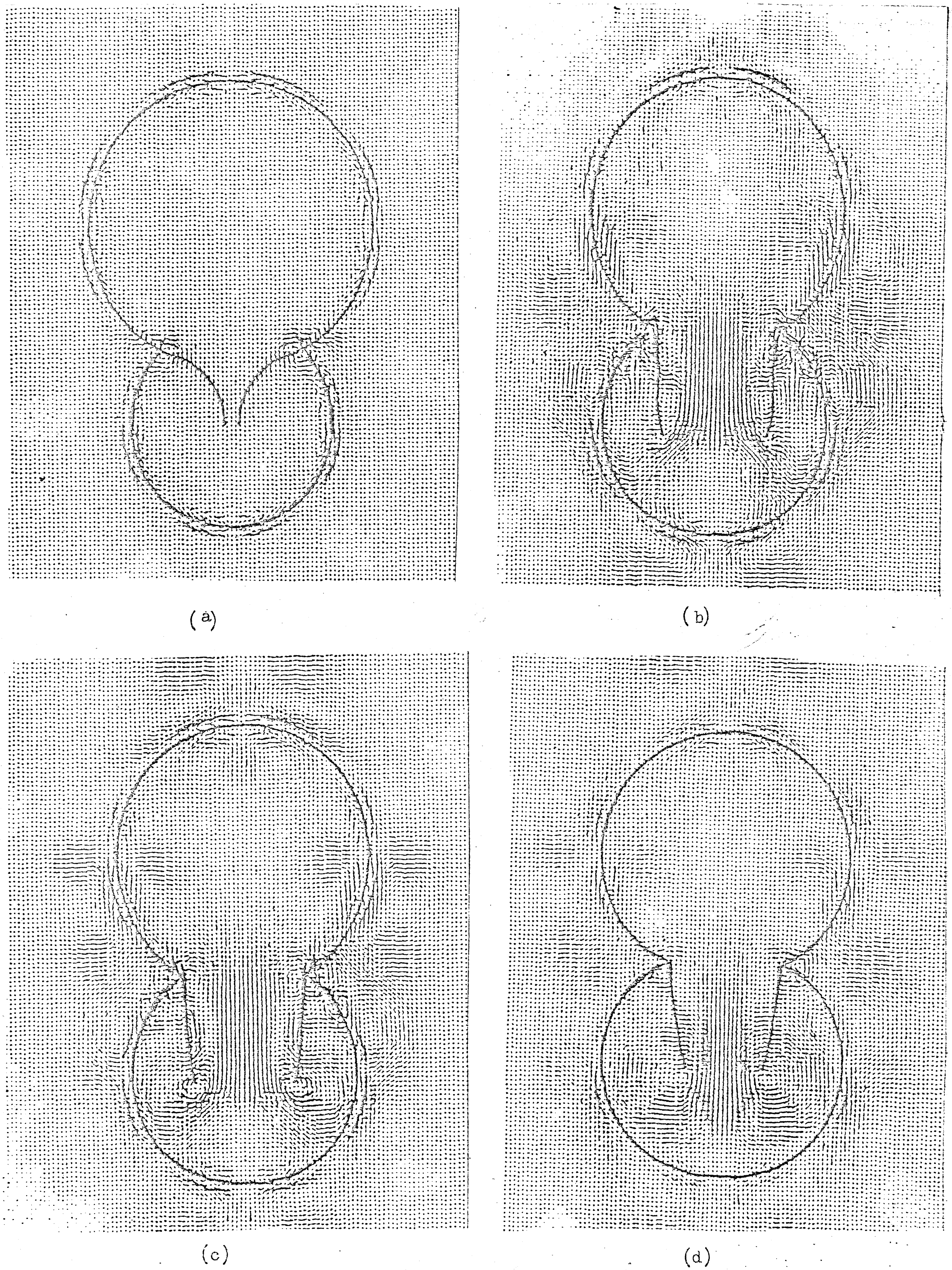
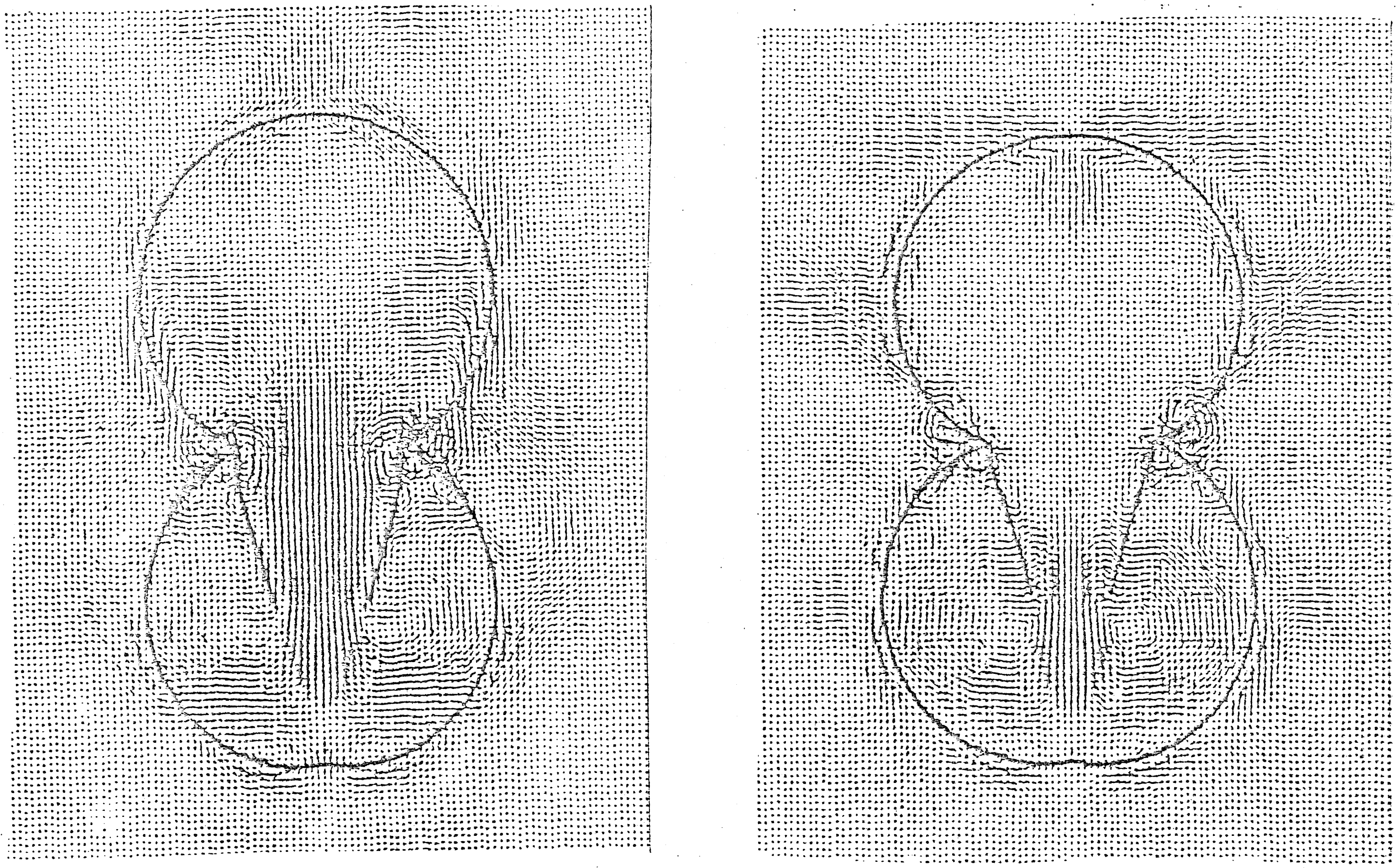


Figure 10



(e) (f)

Figure 10 continued