To ASME

THE APPLICATION OF THE GORLIN EQUATION TO THE STENOTIC MITRAL VALVE

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ABSTRACT

The time averaged equation of motion is used to calculate the area of a stenotic mitral valve. The components of the discharge coefficient are analyzed with respect to the unique configuration of the valve and the surrounding ventricle, and the exact equation is used as the basis of an error analysis of the Gorlin equation. Measurements of transmitral pressures and flow in the dog indicate that despite errors inherent in its derivation and application, the Gorlin equation provides a reasonably good estimate of the stenotic mitral area.

INTRODUCTION

We have shown (1) that pulsatile flow across an orifice is quasi-steady within the physiological range and can be described by the following equation:

$$P_1-P_2 = \rho(C_\delta/C_c)^2\left[1-(\beta C_c)^2\right] Q^2/2A_o^2 \qquad (1)$$

In this formulation, P_2 is the downstream pressure far from the vena contracta. Since conditions *in vivo* make it impossible to locate the downstream pressure tap at the contraction, it becomes necessary to define a loss coefficient, $C_\delta^2 = (P_1-P_2)/(P_1-P_2)$, to represent the fraction of the maximum pressure drop which is not recovered downstream. The contraction coefficient, and the area ratio, are defined in the standard manner: $C_c = A_c/A_o$ and $\beta = A_o/A_1$.

In the cardiac catheterization laboratory, when studying a patient with mitral stenosis, the Cardiologist applies the Gorlin equation (2) which is, in our notation:

$$A_o = \bar{Q} / (0.6)(51.6)\sqrt{\overline{\Delta P}} \qquad (2)$$

Where: $\bar{Q}$ is the mean flow determined from cardiac output, heart rate, and filling time, (the latter is based on the two atrio-ventricular pressure cross-over points); $\overline{\Delta P}$ is the mean pressure gradient between the A-V cross-over points; $51.6 = \sqrt{2g\,\rho_{Hg}/\rho_w}$, and includes the conversion from cm H_2O to mmHg; and 0.6 is the discharge coefficient, C_d. (Note that Gorlin selected 0.7 as the constant, but incorporated the conversion of units in its value, so that effectively his discharge coefficient = 0.6). Putting equation (1) in the same form as equation (2) we get:

$$A_o = \sqrt{Q^2}/C_d(51.6)\sqrt{\Delta P} \qquad (3)$$

where

$$C_d = C_c/C_\delta\sqrt{1-(\beta C_c)^2}$$

In this report we will use the Gorlin equation to calculate the effective orifice area in dogs with mitral stenosis. The inherent errors of this approach and the components of the discharge coefficient will be discussed.

METHODS

Instantaneous A-V pressures and flow in three anesthetized open-chest dogs were measured and recorded as described previously (3). Mitral stenosis was created by suturing together the commissural cusps to create a moderate to tight stenosis (less than 1.0 cm^2). The stenotic area was measured at the time of surgery and again at the termination of the experiment. The oscillographic records were digitized and integrated by programmable calculator to give mean flows and pressure gradients from which orifice areas were calculated.

RESULTS

Figure 1 shows the record from one dog with a measured orifice area of 0.8 cm^2. Beats 1, 2, and 5 are normal, beat 3 is a ventricular premature contraction, and beat 4 is a post VPC beat with a compensatory pause. In this, as in all records, calculations were based on normal and arrhythmic periods and at various heart rates; i.e., under many conditions of flow for a given stenotic area.

The calculated orifice areas, using equation (2), the Gorlin equation, were within an average of 5% of the measured area, with no significant differences or trends during arrhythmias and heart rate changes.

DISCUSSION

There are three sources of error in the use of the Gorlin equation. 1) <u>The square root error</u>: Since the mean of the flow is always less than the square root of the mean of the flow squared (rms), A_o in equation (2) will be underestimated by an amount which depends on the shape of the flow wave-form. The rms value of the positive half of a pure sinusoid, for example, will be 11% greater than its mean. The more blunt wave-form of flow in mitral stenosis would thus require an estimated +5% correction. 2) <u>True filling time</u>: Referring to Figure 1 it can be seen that the diastolic filling period for flow is always greater than the filling period calculated from the pressure crossover points. In mitral stenosis this time delay is approximately 20 msec, suggesting a correction of -5% to the flow. (The percent correction remains relatively invariant with heart rate). 3) <u>True estimate of ΔP</u>: Referring again to Figure 1, we note that during the time required for the flow to decelerate to zero, the ventricular pressure exceeds the atrial pressure, so that the mean pressure gradient should include a negative phase. The pressure gradient in the Gorlin equation is thus overestimated by approximately 5% and is independent of heart rate.

The errors inherent in the Gorlin equation are thus seen to be minor and tend to cancel each other. The favorable clinical experience with the Gorlin equation over the last twenty years, and the favorable experience we have had in the controlled laboratory situation, suggest that the value of 0.6 for the discharge coefficient is reasonably good. This conclusion is also supported by the following reasoning.

Since the contraction coefficient lies between 0.6 and 1.0, and the loss coefficient can never be greater than 1.0, we conclude that the discharge coefficient is indeed 0.6, and that $C_c = 0.6$ and $C_\delta = 1.0$. These are acceptable values because the Reynolds number in the jet is greater than 1000 and because the total inflow to the ventricle has been decelerated to zero velocity by the end of diastole. Thus, there is a significant jet contraction leading to increased kinetic energy in the stenotic jet which is subsequently dissipated in the ventricle. That is, the ventricle is not diffuser-like and does not recover any of the pressure drop in the jet. This explains why patients with moderate mitral stenosis must restrict their physical activity and keep their cardiac output down.

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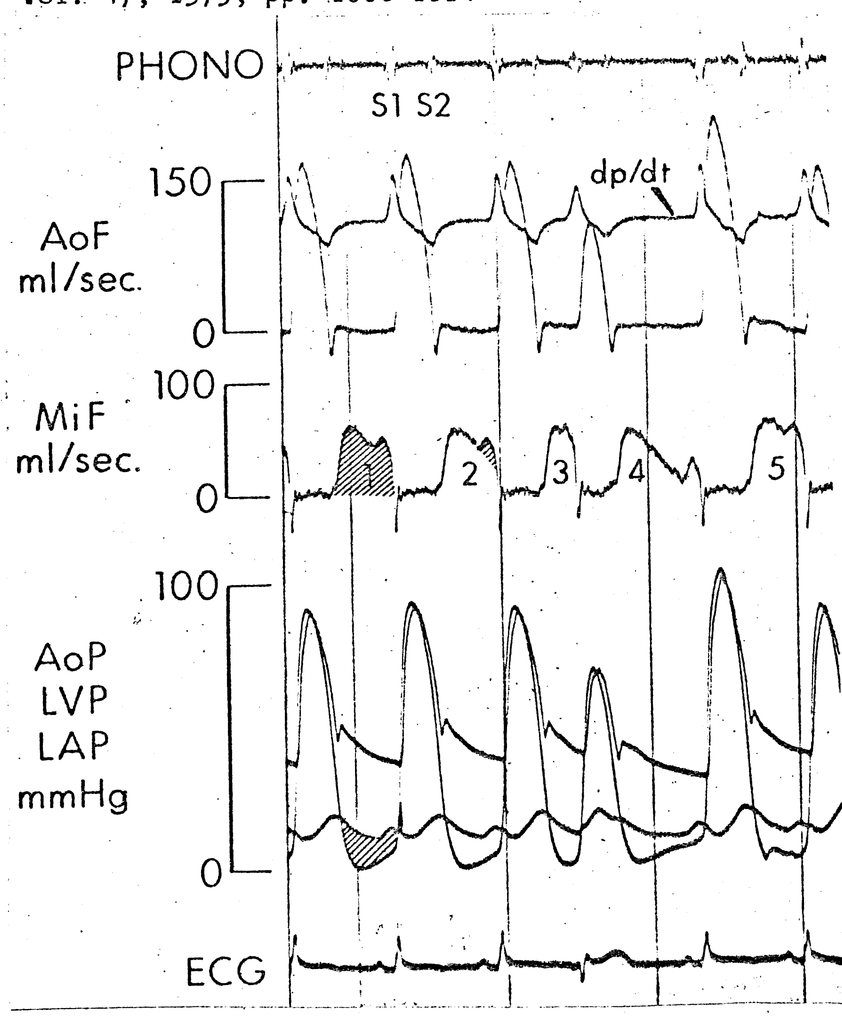

Fig.1 The oscillographic record from a dog with a surgically created acute mitral stenosis. The cross hatched areas in beat 1 indicate the filling volume and its corresponding pressure gradient. From the top down: Phono = phonocardiogram (S1 and S2 are the first and second heart sounds); dp/dt= derivative of left ventricular pressure; AoF = aortic flow; MiF = mitral flow; AoP, LVP, LAP = aortic, left ventricular and left atrial pressure; ECG = electrocardiogram