

TWO-DIMENSIONAL SIMULATIONS OF VALVELESS PUMPING USING THE IMMERSSED BOUNDARY METHOD*

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Abstract. Flow driven by pumping without valves is examined, motivated by biomedical applications: cardiopulmonary resuscitation (CPR) and the human fetus before the development of the heart valves. The direction of flow inside a loop of tubing which consists of (almost) rigid and flexible parts is investigated when the boundary of one end of the flexible segment is forced periodically in time. Despite the absence of valves, net flow around the loop may appear in these simulations. The magnitude and even the direction of this flow depend on the driving frequency of the periodic forcing.

Key words. valveless pumping, immersed boundary method, frequency, CPR

AMS subject classifications. 76D05, 76Z99, 92-08, 92B99, 92C05, 92C50

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1. Introduction. Pumping blood in one direction is the main function of the heart, which is equipped with valves that ensure unidirectional flow. Is it possible, though, to pump blood without valves? This paper is intended to show by numerical simulation the possibility of a net flow which is generated by a valveless mechanism in a circulatory system. Simulations of valveless pumping are motivated by physical experiments of Kilner [12], which had been developed from experiments by Liebau [13, 14, 15]. Kilner observed net flow in one direction which depends on the location of periodic forcing in his experiments. We have examined flows driven by pumping without valves in Liebau’s model, a loop of tubing of which part is almost rigid and the other part is flexible. In agreement with Kilner, we find that net flow can indeed be driven around such a loop by periodic forcing at one location, but we also find something new: the direction of the flow depends on the driving frequency of the periodic forcing.

As reviewed by Moser et al. [18], there have been several earlier investigations of valveless pumping. Harvey (1628), Weber (1834), Donders (1856), and Thomann [26] suggest theories of valveless pumping, and Ozanam (1881) and Liebau [13, 14, 15] make the various types of physical experiments to explain the mechanism of valveless pumping. Moser et al. [18] try to identify the responsible mechanism and conditions under which this mechanism operates. Their proposed mechanism involves a difference in impedance of two pathways between compliant reservoirs.

One of the applications of valveless pumping may turn out to be cardiopulmonary resuscitation (CPR). The blood flow during CPR has been explained by two theories:

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the thoracic pump and cardiac compression mechanisms. In support of the thoracic pump model, it has been reported that the heart is “a passive conduit for blood flow” during chest compression [1, 3, 10], with an open mitral valve throughout the cardiac cycle and antegrade (forward) transmitral blood flow even during chest compression. Werner et al. [27] also report that the mitral valve remains open throughout the entire compression-release cycle of CPR while the aortic valve opens during the compression phase of CPR and closes during the release phase. Thus, the left side of the heart appears to act as a conduit for passage of blood, and mitral valve closure is not necessary for forward blood flow during CPR. Despite the observed lack of valve function, some patients with cardiac arrest are successfully resuscitated by external chest-compression CPR [25]. These findings are controversial, however. Other researchers such as Feneley et al. [4] report results that are inconsistent with the thoracic pump theory and support direct cardiac compression as the primary mechanism of blood flow with the high-impulse manual CPR technique. These investigators favor the cardiac compression theory, in which the heart acts as a pump and its valves function normally. It is possible, of course, that both theories are correct, each in a different set of circumstances. Our computational model of valveless pumping might be applicable to the thoracic pump model and help to understand the thoracic pump mechanism. If the magnitude and even the direction of flow in valveless pumping are indeed frequency dependent, as our results seem to indicate, it is of obvious importance to know what frequency of chest compression will produce the most effective CPR.

Another biological example of valveless pumping may occur in the human embryo at the end of the third week of gestation. At this stage of development, the valves of the heart have not yet formed. Nevertheless, there is a net flow in the circulatory system that is somehow generated by the beating of the heart.

An industrial application of valveless pumping is in microelectromechanical system (MEMS) devices [17], where there is a need to produce fluid motion without moving anything inside the fluid. MEMS devices could be built that incorporate flexible flow channels. In that case, our findings might be applicable to the design of valveless pumps for MEMS devices.

We do not attempt to construct a theory of valveless pumping in this report. Instead, we use numerical simulation as an “experimental” tool to study this mysterious phenomenon.

The rest of the paper is organized as follows. In section 2 the immersed boundary method will be introduced. Section 3 is devoted to the description of the two-dimensional valveless pumping model. The results will be discussed in section 4. In section 5 several special cases will be observed. Finally, some conclusions will be drawn in section 6.

2. The immersed boundary method.

2.1. Mathematical formulation. The immersed boundary method is applicable to problems involving an elastic structure interacting with a viscous incompressible fluid. It has been applied to a variety of problems, particularly in biophysics, including two-dimensional and three-dimensional simulations of blood flow in the heart [19, 20, 9, 21, 22], the design of prosthetic cardiac valves [16], platelet aggregation during blood clotting [7], wave propagation in the cochlea [2], the flow of suspensions [8], peristaltic pumping of solid particles [6], and aquatic animal locomotion [5]. The version of the immersed boundary method used in this work is that of [23], except that here we are in two space dimensions.

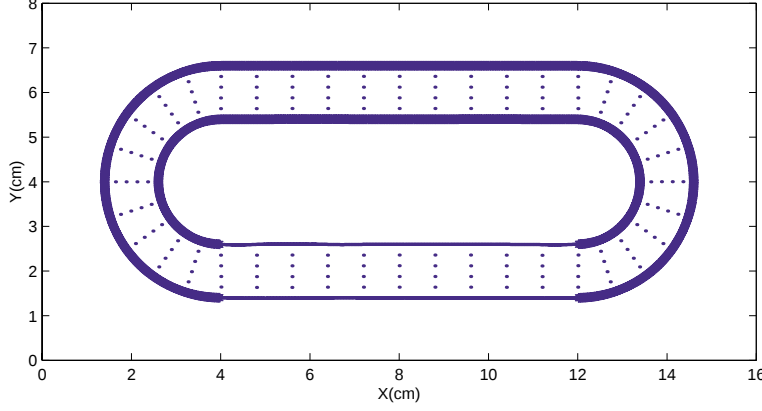


FIG. 1. *Initial position of two-dimensional valveless pumping: flexible boundary (thin lines), almost rigid (thick lines), and fluid markers (dots).*

The philosophy of the immersed boundary method is that the elastic material is treated as a part of the fluid in which singular forces are applied. The fluid and the elastic immersed boundary constitute a coupled mechanical system: The motion of the fluid is influenced by the force generated by the immersed boundary on the fluid, but at the same time the immersed boundary moves at the local fluid velocity and exerts singular forces locally on the fluid. The strength of this method is that it can handle the complicated and time dependent geometry of the elastic immersed boundary which interacts with the fluid, and that it does so while using a fixed regular lattice for the fluid computation.

Consider a viscous incompressible fluid which fills a periodic rectangular box Ω and an immersed boundary S_b in the shape of a racetrack which is contained in the box, where $b = 1$ (inner immersed boundary) or $b = 2$ (outer immersed boundary). Figure 1 displays the two-dimensional model in which we shall simulate valveless pumping.

We shall now consider the mathematical formulation of the equations of motion for the fluid-immersed boundary system. Let ρ be the constant fluid density and μ be the constant viscosity. The equations of motion are then as follows:

$$(2.1) \quad \rho \left(\frac{\partial \mathbf{u}(\mathbf{x}, t)}{\partial t} + (\mathbf{u}(\mathbf{x}, t) \cdot \nabla) \mathbf{u}(\mathbf{x}, t) \right) + \nabla p(\mathbf{x}, t) = \mu \nabla^2 \mathbf{u}(\mathbf{x}, t) + \mathbf{F}(\mathbf{x}, t),$$

$$(2.2) \quad \nabla \cdot \mathbf{u}(\mathbf{x}, t) = 0,$$

$$(2.3) \quad \mathbf{F}(\mathbf{x}, t) = \int_{S_b} \mathbf{f}_b(s, t) \delta^2(\mathbf{x} - \mathbf{X}_b(s, t)) ds,$$

$$(2.4) \quad \mathbf{U}_b(s, t) = \int_{\Omega} \mathbf{u}(\mathbf{x}, t) \delta^2(\mathbf{x} - \mathbf{X}_b(s, t)) d\mathbf{x},$$

$$(2.5) \quad \frac{\partial \mathbf{X}_b(s, t)}{\partial t}(s, t) = \mathbf{U}_b(s, t),$$

$$(2.6) \quad \mathbf{f}_b(s, t) = -(\kappa_t)_b(\mathbf{X}_b(s, t) - \mathbf{Z}_b(s, t)) + \kappa_c \left(\frac{\partial^2 \mathbf{X}_b(s, t)}{\partial s^2} \right).$$

Equations (2.1) and (2.2) are the fluid (Navier–Stokes) equations in Eulerian form. The fluid velocity $\mathbf{u}(\mathbf{x}, t)$, fluid pressure $p(\mathbf{x}, t)$, and singular force density $\mathbf{F}(\mathbf{x}, t)$ are unknown functions of $(\mathbf{x}, t)$, where $\mathbf{x} = (x, y)$ are fixed Cartesian coordinates and t is the time. Equations (2.5) and (2.6) are the immersed boundary equations in Lagrangian form. The configurations of the immersed boundaries are described by the unknown functions $\mathbf{X}_b(s, t)$, where $b = 1$ (inner immersed boundary) or 2 (outer immersed boundary), and $0 \leq s \leq L_1$ for the inner immersed boundary and $0 \leq s \leq L_2$ for the outer immersed boundary. L_1 and L_2 are the unstressed lengths of the inner and outer boundaries, respectively. The boundary force densities $\mathbf{f}_b(s, t)$ and the boundary velocities $\mathbf{U}_b(s, t)$, for the inner and outer boundaries $b = 1$ and $b = 2$, are also unknown functions of s and t . A fixed value of the Lagrangian parameter s marks a material point of the immersed boundary.

In (2.6) the boundary force is computed as a sum of two terms. In the first term, the given function $\mathbf{Z}_b(s, t)$ is called the *target position* of the immersed boundary. This first term provides a restoring force that keeps the boundary points near their target positions. Target positions are used for two purposes in this work: first to maintain the shape of the flow loop, and second, by allowing $\mathbf{Z}_b(s, t)$ to change with time, to apply periodic forcing to the immersed boundaries. The curvature term in (2.6) models an elastic membrane under tension. Together, the two terms model the boundaries as tethered elastic membranes.

Note that the curvature term actually is the response to stretching of the membrane; it is not a response to curvature per se. It can be derived from an energy function which is just the sum of the squares of the lengths of the individual segments. Also, the curvature term has little effect on the parts of the racetrack that are stiffly pinned to target points, since those forces dominate there. It is important only on the straight flexible segment.

Equations (2.4) and (2.5) are, in effect, the no-slip condition, since they state that the immersed boundary moves at the local fluid velocity. Equations (2.3) and (2.4) are the interaction equations in mixed Eulerian and Lagrangian form. The core of the immersed boundary method is the delta function, which describes the interaction between the fluid and the immersed boundary. Both of the interaction equations are in integral form with kernel $\delta^2(\mathbf{x} - \mathbf{X}_b(s, t))$. The integral in (2.4) is taken over the two-dimensional space occupied by the fluid. However, in (2.3), the integral is taken over a one-dimensional space, the immersed boundary, but the delta function is a product of two one-dimensional delta functions: $\delta^2(\mathbf{x}) = \delta(x)\delta(y)$. Therefore, $\mathbf{F}(\mathbf{x}, t)$ is a singular force density. The total force is finite despite the singularity in the force density $\mathbf{F}(\mathbf{x}, t)$.

2.2. Numerical method. In this section, we present the summary of the immersed boundary method to find a numerical solution to the system of equations (2.1)–(2.6). For details of this numerical method, see [11, 23]. Let superscripts and subscripts denote the time step index and the spatial discretization, respectively. Let the time proceed in steps of duration Δt , let Δx and Δy be the fluid-lattice spacing, and let Δs_b be the unstressed distance between material points of the immersed boundary. The fluid equations (2.1) and (2.2) in Eulerian form are discretized on a fixed rectangular lattice at time $t = n\Delta t$: $\mathbf{x}_{jk}^n = \mathbf{x}(j\Delta x, k\Delta y, n\Delta t)$, where $j = 0, \dots, N_x - 1$, $k = 0, \dots, N_y - 1$, and $n = 0, 1, \dots$. The immersed boundary equations (2.5) and (2.6) in Lagrangian form are discretized on a collection of moving

points in the immersed boundary at time $t = n\Delta t$: $\mathbf{X}_{bl}^n = \mathbf{X}_b(l\Delta s_b, n\Delta t)$ which do not coincide with the fluid lattice, where $l = 1, \dots, M_1$ and $L_1 = M_1\Delta s_1$ for $b = 1$ (inner boundary) or $l = 1, \dots, M_2$ and $L_2 = M_2\Delta s_2$ for $b = 2$ (outer boundary).

Our goal is to compute the update $\mathbf{u}^{n+1}$, $\mathbf{X}_b^{n+1}$ from given $\mathbf{u}^n$, $\mathbf{X}_b^n$. This is done as follows:

For simplicity, let $N_x = N_y = N$ and choose $\Delta x = \Delta y = h$.

Step 1. Find the force $\mathbf{f}_b^n$ on the immersed boundary from the given boundary configuration $\mathbf{X}_b^n$.

For $l = 1, \dots, M_b$, and $b = 1$ (inner boundary) or 2 (outer boundary),

$$(2.7) \quad \mathbf{f}_{bl}^n = -(\kappa_t)_l(\mathbf{X}_{bl}^n - \mathbf{Z}_{bl}^n) + \kappa_c \left(\frac{\mathbf{X}_{b(l+1)}^n - 2\mathbf{X}_{bl}^n + \mathbf{X}_{b(l-1)}^n}{\Delta s_b^2} \right),$$

where $\mathbf{Z}_b^n$ is a target position at $t = n\Delta t$, Δs_b is an arc length, κ_t is a stiffness constant, and κ_c is another stiffness constant for the curvature force term.

Note that the subscript arithmetic on l in (2.7) has to be interpreted in a periodic sense, since the boundary is closed: when $l = M_b$, $l + 1 = 1$; when $l = 1$, $l - 1 = M_b$. The target positions $\mathbf{Z}_b^n$ are calculated by the given boundary configuration $\mathbf{X}_b^n$. The formulations of the target positions will be given in the following section.

Step 2. Spread the boundary force into the nearby lattice points of the fluid using the δ function.

$$(2.8) \quad \mathbf{F}_{jk}^n = \sum_{b=1}^2 \sum_{l=1}^{M_b} \mathbf{f}_{bl}^n \delta_h^2(\mathbf{x}_{jk} - \mathbf{X}_{bl}^n) \Delta s_b \quad \text{for } j, k = 0, 1, \dots, N-1,$$

where $\mathbf{x}_{jk} = (jh, kh)$ and δ_h^2 is a smoothed approximation to the two-dimensional Dirac delta function:

$$\delta_h^2(\mathbf{x}) = \frac{1}{h^2} \phi(x/h) \phi(y/h),$$

where

$$\phi(r) = \begin{cases} \frac{3-2|r|+\sqrt{1+4|r|-4r^2}}{8} & \text{if } |r| \leq 1, \\ \frac{5-2|r|-\sqrt{-7+12|r|-4r^2}}{8} & \text{if } 1 \leq |r| \leq 2, \\ 0 & \text{if } 2 \leq |r|. \end{cases}$$

The motivation for this particular choice of $\phi(r)$ is given in [22].

Step 3. Solve the Navier–Stokes equations on the rectangular lattice to get the update $\mathbf{u}^{n+1}$ and p^{n+1} from $\mathbf{u}^n$ and $\mathbf{F}^n$. The periodic boundary conditions for the computational domain are imposed. These equations are solved by the following implicit first order scheme in time and space:

$$(2.9) \quad \rho \left(\frac{\mathbf{u}^{n+1} - \mathbf{u}^n}{\Delta t} + \mathbf{u}^n \cdot \nabla_h^\pm \mathbf{u}^n \right) + \mathbf{D}^0 p^{n+1} = \mu \Delta_h \mathbf{u}^{n+1} + \mathbf{F}^n,$$

$$(2.10) \quad \mathbf{D}^0 \cdot \mathbf{u}^{n+1} = 0.$$

The difference operators in these equations are constructed as follows. First, the forward (D^+), backward (D^-), and centered (D^0) difference operators are defined in the standard way. Then $\mathbf{D}^0$ is defined by $\mathbf{D}^0 = (D_x^0, D_y^0)$. This is used in the

discrete divergence and gradient. Next, the discrete Laplacian for the viscous term is defined by $\Delta_h \mathbf{u} = \sum_{\alpha=1}^2 D_\alpha^+ D_\alpha^- \mathbf{u}$. Finally, the upwind difference operator $\mathbf{u} \cdot \nabla_h^\pm = \sum_{\alpha=1}^2 u_\alpha D_\alpha^\pm$, where

$$u_\alpha D_\alpha^\pm = \begin{cases} u_\alpha D_\alpha^+ & \text{if } u_\alpha < 0, \\ u_\alpha D_\alpha^- & \text{if } u_\alpha > 0. \end{cases}$$

This difference scheme is stable (provided that $\sum_{\alpha=1}^2 |u_\alpha| \Delta t < h$) because of the choice of the upwind scheme for the convection terms and the backward Euler differencing used for the Stokes system.

Because of the periodic boundary conditions of the computational domain, it is natural to use the fast Fourier transform (FFT) algorithm to solve (2.9) and (2.10) for the unknowns $(\mathbf{u}^{n+1}, p^{n+1})$. To get the update $\mathbf{u}^{n+1}$ and p^{n+1} , first take the discrete Fourier transformation of (2.9) and (2.10). Then, the system can be solved for $\hat{u}_{lm}$, $\hat{v}_{lm}$, and $\hat{p}_{lm}$ for each l and m , $0 \leq l, m \leq N-1$. Finally, evaluate $\mathbf{u}^{n+1}$ and p^{n+1} by applying the inverse FFT algorithm to $\hat{\mathbf{u}}^{n+1}$ and $\hat{p}^{n+1}$.

Step 4. Once the updated fluid velocity, $\mathbf{u}^{n+1}$, has been determined, we can find the velocity, $\mathbf{U}_b^{n+1}$, and then the new position, $\mathbf{X}_b^{n+1}$, of the immersed boundary points. This is done using a discretization of (2.4) and (2.5).

The difference approximations to the interpolation equation and no-slip condition are expressed as follows:

For $l = 1, \dots, M_b$, and $b = 1$ (inner boundary) or 2 (outer boundary),

$$(2.11) \quad \mathbf{U}_{bl}^{n+1} = \sum_{j,k=0}^{N-1} \mathbf{u}_{jk}^{n+1} \delta_h^2(\mathbf{x}_{jk} - \mathbf{X}_{bl}^n) h^2,$$

$$(2.12) \quad \mathbf{X}_{bl}^{n+1} = \mathbf{X}_{bl}^n + \Delta t \mathbf{U}_{bl}^{n+1}.$$

Note that we use the same delta function in (2.11) as the one in the interaction equation for the force term, (2.8).

This completes the description of the process (Steps 1–4, above) by which the quantities $\mathbf{u}$ and $\mathbf{X}$ are updated.

3. Two-dimensional model of valveless pumping. In this section, we shall introduce a two-dimensional computational model of valveless pumping. The initial configuration of our model is presented first. Then we present the motions of target positions to investigate fluid motions around the flow loop. In particular, we explain how the time dependent target positions are used to provide the periodic forcing which is applied on the one end of the immersed boundary. Finally, we display the physical and computational parameters which are used in our numerical experiments.

3.1. Initial position. Consider an incompressible viscous fluid with a constant density ρ and viscosity μ in a periodic rectangular box which contains an immersed elastic boundary. Figure 1 shows the initial configuration of the immersed boundary of two-dimensional valveless pumping in our numerical experiments. In this two-dimensional model, the immersed boundary consists of two closed curves, each in the form of a racetrack. The part of each curve shown with thick lines in Figure 1 is almost rigid and the other part with thin lines is flexible. The fluid fills the entire box. Fluid markers, however, are only shown inside the flow loop, since that is the region of interest.

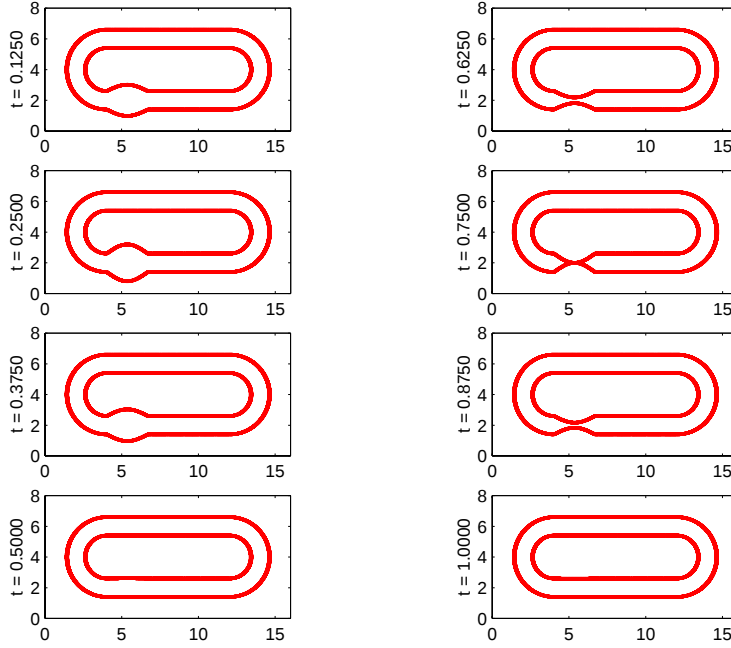


FIG. 2. *The target positions during one cycle: The motion of fluid inside the loop is driven by the periodic vertical expansion and contraction of the target configuration of the tube. This motion is confined to the left 1/3 of the flexible segment of tubing.*

Throughout this paper, we assume that the motions are driven by periodic vertical oscillations of the left 1/3 of the flexible tube boundary. Details of how these oscillations are applied will be described next.

3.2. Target positions and parameters. Recall that the equation for the force on the immersed boundary (2.6) involves target positions $\mathbf{Z}_b(s, t)$. After discretization, these become $\mathbf{Z}_{bl}^n$. For most of the flow loop, these are independent of time and serve the purpose of maintaining the racetrack shape of the flow loop. Time dependent target positions are used in the left 1/3 of the flexible segment of the flow loop (as shown in Figure 2) in order to provide periodic forcing to the flow. Figure 2 displays the target positions at eight equally spaced times over one cycle.

Now we describe the mathematical formulations of the time dependent target positions in the left 1/3 of the flexible segment of tubing.

Let $\mathbf{Z}_b(s, t) = (Z_{xb}(s), Z_{yb}(s, t))$, where s is restricted to the range of values that defines the left 1/3 of the flexible segment of tubing. This may be a different range of s values in the case $b = 1$ (inner boundary) than in the case $b = 2$ (outer boundary). Note that the x component of the target position $\mathbf{Z}_b(s, t)$ is independent of t , whereas the y component varies with time in the manner that we prescribe.

Define

$$A(s, t) = A_0 \sin\left(\frac{2\pi t}{T}\right) \sin\left(\pi \frac{Z_{xb}(s) - 0.25X_{scale}}{\frac{1}{3}0.5X_{scale}}\right),$$

where A_0 is the amplitude of the target position motion, T is its period, and X_{scale} is the length of the computational domain (i.e., its size in the x direction).

TABLE 1
Physical parameters.

Physical parameters	Symbol	
Density	ρ	1 g/cm ³
Viscosity	μ	0.01 g/cm · s
Circumference of a loop	D	28.57 cm
Diameter of tube	d	0.6 cm
Computational domain	$X_{scale} \times Y_{scale}$	16 cm × 8 cm
Period	T	0.05 s ~ 4 s
Amplitude(target)	A_0	0.4 cm and 0.6 cm
Duration of experiment	t_{max}	150 s
Stiffness constant(almost rigid)	κ_t	26000 g/s ² · cm
Stiffness constant(flexible)	κ_t	900 g/s ² · cm
Stiffness constant(curvature)	κ_c	120 g · cm/s ²

TABLE 2
Computational parameters.

Computational parameters	Symbol	
Fluid lattice	$N_x \times N_y$	256 × 128
Number of immersed boundary points	$M_1 + M_2$	3654
Meshwidth	$h = \Delta x = \Delta y$	0.0625 cm
Initial distance between boundary points	$\Delta s_1 = \Delta s_2$	$h/4 = 0.0156$ cm
Time step duration	Δt	$0.5 h^2 = 0.00195$ s

The flexible segment begins at $x = 0.25X_{scale}$ and ends at $x = 0.75X_{scale}$. Thus, the whole flexible segment has length $0.5X_{scale}$, and the part of it in which we allow the target position to move has length $\frac{1}{3}0.5X_{scale}$.

With $A(s, t)$ defined as above, let $Z_{by}(s, t)$ be defined as follows:

$$Z_{by}(s, t) = \begin{cases} 0.25Y_{scale} + 0.5d + A(s, t) & \text{if } b = 1 \text{ (inner boundary),} \\ 0.25Y_{scale} - 0.5d - A(s, t) & \text{if } b = 2 \text{ (outer boundary),} \end{cases}$$

where d is the resting diameter of the tube, Y_{scale} is the width of the computational domain (i.e., its size in the y direction), and s is again restricted to the range of values that defines the left 1/3 of the flexible segment of tubing.

The flows that we have investigated are all driven by these periodic motions of the time dependent target positions in the left 1/3 of the flexible segment of tubing.

Note that the target tube, taken as a whole, has nonconstant volume. This is reasonable, since the physical tube is only connected to the target tube by springs and does not follow the target motion in detail; see Figures 10, 11, and 12. In fact, the volume conservation of the physical tube is valid (see Figures 4 and 5) and does not reflect the periodic volume changes imposed on the target tube.

In this work, we use CGS units, but to give a sense of the dimensionless character of the flow, we sometimes report results in terms of a Reynolds number, which is defined by $Re = \frac{\rho U d}{\mu} = \frac{\rho \Phi}{\mu}$, where U is a time-averaged velocity, d is a diameter of the tube, ρ is a constant density, μ is viscosity, and Φ is a time-averaged flux. Note that this Reynolds number refers to the time-averaged velocity and flux, so any nonzero value indicates that valveless pumping has occurred. The Reynolds number, so defined, has varied in our computations between 0 and about 160.

Tables 1 and 2 display the physical and computational parameters, respectively. As indicated in Table 1, the two physical parameters that we systematically vary

TABLE 3
The ratios of the L_2 difference of velocities.

L_2 difference ratio	
$\ u_{64}-u_{128}\ _2/\ u_{128}-u_{256}\ _2$	1.910726
$\ u_{128}-u_{256}\ _2/\ u_{256}-u_{512}\ _2$	1.995359
$\ v_{64}-v_{128}\ _2/\ v_{128}-v_{256}\ _2$	1.662576
$\ v_{128}-v_{256}\ _2/\ v_{256}-v_{512}\ _2$	1.869106

in this work are the period and amplitude of the prescribed motion of the target positions. The resulting flows are definitely dependent on these two parameters. In particular, they determine not only the amount of net flow that develops but even the direction of the net flow around the loop.

4. Results and discussion. The two main results of this paper are as follows: First, a net flow around the loop is produced by the periodic forcing on one end of the flexible boundary, despite the absence of valves. Previous investigators have observed this phenomenon in physical experiments [12], and a theoretical explanation based on a lumped parameter (ODE) model has been proposed [18], but this is the first time, to our knowledge, that valveless pumping has been demonstrated by a computer simulation based on the Navier–Stokes equations. Second, we find that the direction of flow around the loop is determined not only by the position of the periodic compression (as in [12]) but also by the amplitude and frequency of the driving force. This is a new, unexpected phenomenon, not previously reported, and the most important prediction of our model.

Preliminary physical experiments performed in the Courant Institute WetLab confirm that the flow can be driven in either direction from the same location depending on the details of the forcing. Experiments will be described in a future publication with Jun Zhang.

In this section, we first justify our numerical method and then show that the amplitude and the frequency are the crucial parameters to determine the direction of a net flow around the loop. Some special cases of valveless pumping are then discussed.

4.1. Checks on the numerical method. We report two checks on the validity of the numerical method. One check on the computation is to show that our numerical scheme has first order accuracy in time and space. Another check is to see whether the volume of the closed flow loop is conserved.

Numerical convergence. To test accuracy of our numerical scheme, we perform the same computation on the successive lattice refinements and compare the results in the L_2 norm. The physical parameters of this computation are as follows: period = 1.55 s, amplitude of the target position = 0.6 cm, and number of cycles = 96. The other physical parameters are the same as ones in Table 1. We consider the three successive mesh sizes within a fixed size physical domain: $N_x \times N_y = 128 \times 64$, 256×128 , and 512×256 . The ratio of the time step duration to the meshwidth is kept fixed throughout this study: $\Delta t/\Delta x = 0.0312$ s/cm. As N_x and N_y vary, the number of points on the immersed boundary changes in proportion to N_x or N_y , which, of course, are changing in proportion to each other. Specifically, we choose an initial distance between immersed boundary points which is equal to $\Delta x/4$.

Table 3 shows the results of the numerical convergence. The ratio of the L_2 norms on the difference of velocities, $\mathbf{u} = (u, v)$, at the successive lattice refinements

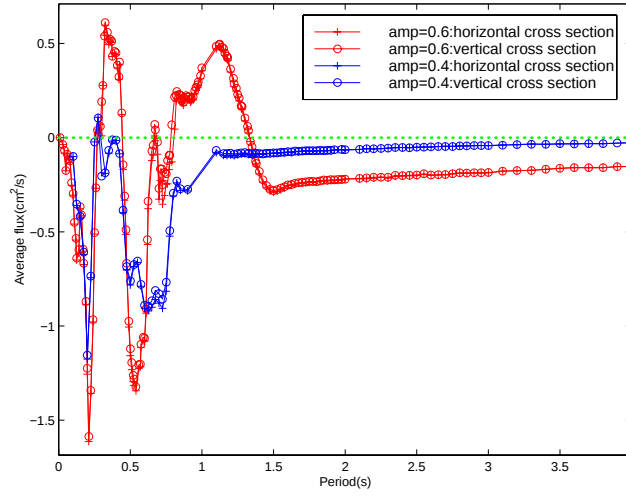


FIG. 3. Average flow versus period (1/frequency): Flows with two different amplitudes of target positions, $A_0 = 0.6$ cm and $A_0 = 0.4$ cm, are compared. The plus data points denote fluxes computed on a horizontal cross section through the middle of the curved segment of tubing on the right side of the racetrack, and the circle data points denote fluxes computed on a vertical cross section through the middle of the straight segment of tubing at the top of the racetrack. In both cases, the time-averaged flux is plotted as a function of the period of the imposed oscillation in target position that drives the flow. Each pair of data points summarizes a separate numerical experiment, the duration of which is 150 s. Positive flux denotes clockwise net flow around the loop of tubing; negative flux denotes counterclockwise net flow. The existence of net flow in these numerical experiments is evidence of valveless pumping. This figure shows that the frequency is a crucial factor to determine the direction and magnitude of flow, and also shows the conservation of volume (area) by the comparison of time-averaged fluxes at two locations.

are compared in Table 3. Since the asymptotic ratio is almost 2, our numerical method has almost first order accuracy. Presumably, the numbers in the table are converging to 2, but it would take computations on finer grids to show this.

Conservation of volume (area). The volume (area) should be conserved in time, since the fluid is incompressible. The conservation of volume (area) is checked in the following two ways: First, the time-averaged flux on two different cross sections of flow loop are compared. Second, the area inside the flow loop is computed as a function of time to see how much it varies. The time-averaged flux is defined by the mean flux computed on a cross section through the middle of the curved (or straight) segment of tubing on the racetrack over the simulated time.

Figure 3 displays the time-averaged flux, which is the main output of our numerical experiments concerning valveless pumping, plotted as a function of the period of the imposed oscillation in target position that drives the flow. To check volume conservation, these fluxes have been computed on two different cross sections of the tube: a vertical cross section in the middle of the straight segment that forms the top of the tube and a horizontal cross section in the middle of the curved segment of tubing on the right. Two different amplitudes of the target positions, $A_0 = 0.6$ cm and $A_0 = 0.4$ cm, are chosen. The time-averaged fluxes at each of these two cross sections practically coincide, over a wide range of periods. (Figure 3 contains four plots but appears to contain only two because the agreement of flows measured at different cross sections is so good.) Other physical and numerical parameters are as

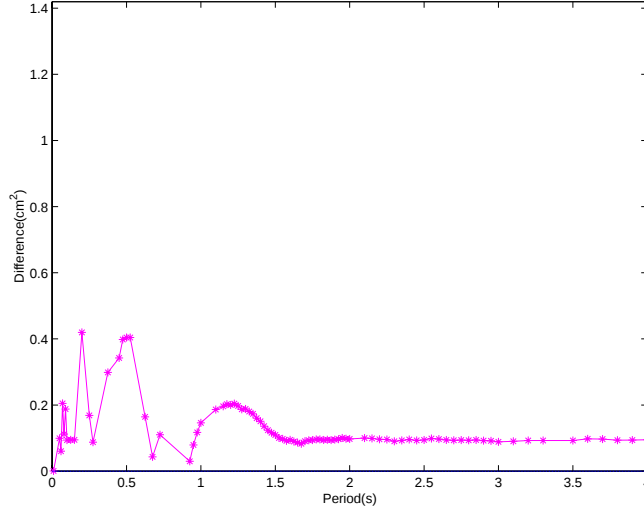


FIG. 4. *Conservation of volume (area): The difference between the time-averaged area and the initial area inside the loop versus the period of the driving oscillation. Each data point summarizes a different numerical experiment of 150 s duration (simulated time). The area of the loop is 34.2796 cm^2 initially. The maximum error occurs at a driving period of 0.2 s, and is equal to 0.4196 cm^2 , which is 1.22% of the initial area.*

shown in Tables 1 and 2.

Figure 4 shows the difference between the time-averaged area inside the flow loop and the initial area inside the flow loop plotted as a function of the period of the driving oscillation. Each data point summarizes a different numerical experiment of 150 s duration (simulated time). The parameters are also given in Tables 1 and 2 except the amplitude of the driving oscillation, which is 0.6 cm. The initial area of the flow loop is 34.2796 cm^2 . The maximum difference between the time-averaged area and the initial area occurs at period 0.2 s, and it is only 0.4196 cm^2 , which is 1.22% of the initial area inside the flow loop.

As a further check on the volume (area) conservation, we plot the area within the flow loop as a function of time. This is done for only one case, the driving period of 0.2 s, at which the maximum difference between the time-averaged area and the initial area occurs. Even in this worst case, the area as a function of time is nearly constant; see Figure 5, and note the expanded scale of the plot.

The volume errors we observed in this subsection are judged to be acceptable, but it could be further reduced if desired by using the method of Peskin and Printz [24].

4.2. The time-averaged flow around the loop as a function of the amplitude of the driving oscillation. In this section, we investigate the influence of the amplitude of the driving oscillation (i.e., the amplitude of the prescribed target position motion) on the magnitude and direction of the net flow around the loop of simulated tubing. Four different periods of the driving oscillation are considered: $T = 0.3 \text{ s}$, 0.375 s , 0.525 s , and 1.7 s . Figure 6 displays the time-averaged flux as a function of the amplitude of the driving oscillation at these four chosen periods. Other parameters besides amplitude and period are given in Tables 1 and 2. Positive flow values denote clockwise net flow, and negative values denote counterclockwise net flow.

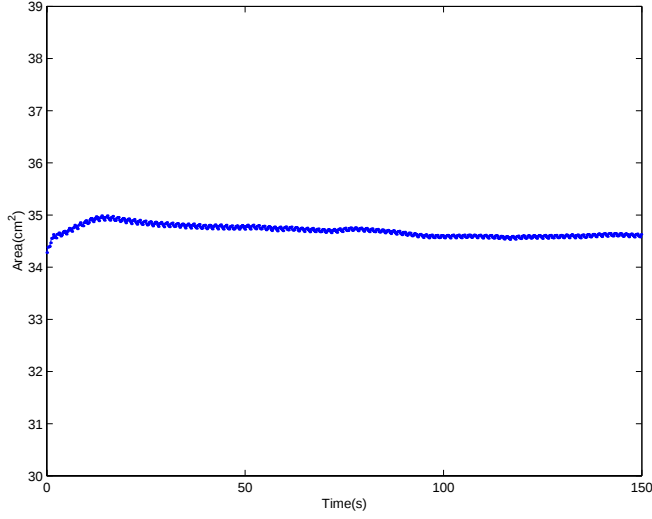


FIG. 5. *Conservation of volume (area): Area versus time at a driving period of 0.2 s. The area inside the flow loop is almost constant in time (note the expanded scale of the plot: 0 is way offscale) even though the chosen period of 0.2 s is the one at which the maximum difference between the time-averaged area and the initial area occurs.*

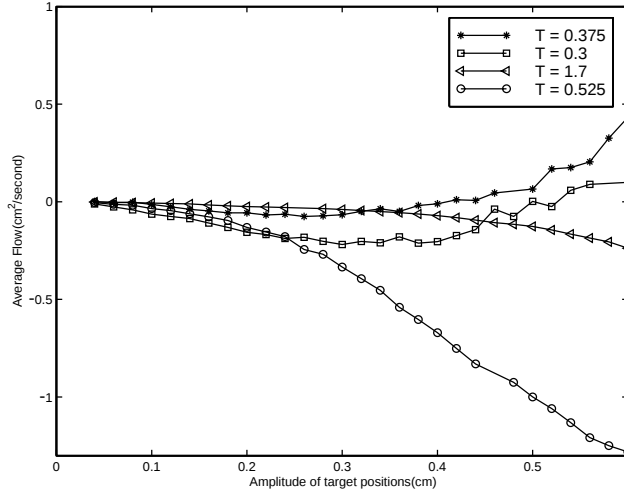


FIG. 6. *Average flux versus amplitude: The flows at four different periods, $T = 0.3$ s, 0.375 s, 0.525 s, and 1.7 s, are compared. Each data point summarizes a separate numerical experiment of 150 s duration (simulated time). For all periods shown, the flow is negative (counterclockwise) at low amplitude, but for some periods ($T = 0.3$ s and $T = 0.375$ s) it reverses and becomes positive (clockwise) at high amplitude.*

The results plotted in Figure 6 show the following features:

- At low amplitude of the driving oscillation, net flow is always in the counterclockwise direction. Its magnitude at any given amplitude depends on the period of the driving oscillation. Of the four examples given in the figure, the periods $T = 0.375$ s and $T = 1.7$ s result in only weak counterclockwise flow,

whereas $T = 0.3$ s and $T = 0.525$ s result in much stronger counterclockwise flow.

- As the amplitude increases a qualitative distinction between the different cases appears. For $T = 0.525$ s and $T = 1.7$ s, the counterclockwise flow simply gets stronger monotonically as the amplitude of the driving oscillation increases. But for $T = 0.3$ s and for $T = 0.375$ s, the flow changes direction at some critical amplitude and becomes clockwise at high amplitude. Note that one cannot predict from the strength of the counterclockwise flow at low amplitude which of the cases will have clockwise flow at high amplitude: Of the two cases that have clockwise flow at high amplitude, one had a strong counterclockwise flow and the other had a weak counterclockwise flow at low amplitude.

Overall, there seems to be a preference for counterclockwise flow in these results. All periods generate counterclockwise flow at low amplitude, and only some periods generate flows that reverse and become clockwise at high amplitude. We can speculate on the reason for this, as follows. Recall that the driving oscillation is imposed at the left end of the flexible segment of tubing, which forms the lower straight segment of the racetrack; see Figure 2. If waves propagate from this source to the right along the flexible segment, these would tend to generate counterclockwise flow by a peristaltic mechanism. This argument leaves open the question of why the flows reverse and become clockwise, for some periods of the driving oscillation, when the amplitude of the driving oscillation becomes sufficiently large.

4.3. The time-averaged flow around a loop as a function of frequency.

In this subsection, we present a new, unexpected phenomenon which is the most important prediction of our model: the driving frequency (1/period) is a crucial parameter to determine the magnitude and even the direction of a net flow generated by valveless pumping.

The time-averaged flow around a loop as a function of the period of the driving oscillation is investigated for two different amplitudes of the driving oscillation, $A_0 = 0.4$ cm and $A_0 = 0.6$ cm. In Figure 3, we plot the time-averaged flux versus period for these two cases. Each data point is the result of a separate numerical experiment, and the parameters are the same as in Tables 1 and 2. As before, positive flow is clockwise, and negative flow is counterclockwise.

The result that is obvious from a glance at Figure 3 is that valveless pumping has a strong dependence on the frequency of the driving oscillation. Indeed, there appear to be resonances at rather specific frequencies, which are most effective in driving the flow in one direction or the other. At the lower amplitude, the net flow is almost always counterclockwise, so these peaks are in the negative direction. As we shift to the higher amplitude, the negative peaks seem to be preserved, but now positive peaks emerge as well. Another indication of the dynamic character of valveless pumping is that it seems to disappear at the extremes of frequency. In the high-frequency (low-period) limit, it is clear from Figure 3 that the net flow approaches zero. This also seems to be true in the low-frequency (high-period) limit, although for the higher amplitude data one cannot be sure whether the net flux is approaching zero or some negative value. In any case, strong valveless pumping happens at specific frequencies that are neither too large nor too small.

Since the driving frequency is an important parameter of valveless pumping, it may be of interest to interpret our results in terms of the Womersley number $Wo = d\sqrt{\omega/\nu}$, where d is the tube diameter (0.6 cm), ν is the kinematic viscosity ($\nu = \mu/\rho =$

$0.01 \text{ cm}^2/\text{s}$), and ω is the driving frequency in radians/s ($\omega = 2\pi/T$). For example, the maximum average clockwise flow occurs at a period of $T = 0.325 \text{ s}$, which is a Womersley number of $Wo = 26$, and the maximum average counterclockwise flow occurs at a period of $T = 0.21 \text{ s}$, corresponding to a Womersley number of $Wo = 33$. In both cases, the Womersley number is substantially larger than 1, which means that the velocity profile is far from parabolic.

5. Case studies. In this section, several special cases of valveless pumping are studied. Recall Figure 3 in the previous section. We chose the special cases based on the results from that figure. The amplitude of the driving oscillation, $A_0 = 0.6 \text{ cm}$, is chosen, since a qualitative distinction between the different cases appears as the amplitude increases, and $A_0 = 0.6 \text{ cm}$ is large enough to show that distinction. The following three cases are considered:

- Maximum average clockwise flow ($T = 0.325 \text{ s}$).
- Almost zero flow ($T = 1.34 \text{ s}$).
- Maximum average counterclockwise flow ($T = 0.21 \text{ s}$).

As before, the fluid motions are driven by the oscillations in target positions which are imposed along the 1/3 left end of the flexible segment of tubing, which forms the lower straight segment of the racetrack. The parameters are as given in Table 1 and 2.

These three cases have been investigated and compared qualitatively in the following ways.

- The angles from the center of the computational domain, $(x, y) = (8 \text{ cm}, 4 \text{ cm})$, to the current positions of the fluid markers inside the flow loop are measured in order to determine the direction of the flow.
- Flowmeter fluxes computed on the vertical cross section through the middle of the straight segment of tubing at the top of the racetrack as functions of time are measured to test whether the fluid motion is in a periodic steady-state through the final duration, $t_{max} = 150 \text{ s}$, and to show the nature of the oscillation and the net progress of the fluid motions.
- The wave motions along the top of the flexible boundaries over one cycle of the periodic steady-state are investigated in order to determine whether the motion looks like a traveling wave or a standing wave (or some other more complicated kind of wave motion).
- The target positions and the real physical positions of the immersed boundary, in particular the flexible segment, are compared in order to see how the time dependent target positions affect the motions of the real physical boundary.
- The velocity vector fields and pressure contours of the maximum clockwise and the maximum counterclockwise cases at 4 different phases over one period after the periodic steady-state are presented.
- Changing the direction of the flow by changing the period *during* a computer experiment.
- Zero net flux for the symmetric driving force.

5.1. Three cases. Angle. Here we examine the net progress of the flow by following the angular position of selected fluid markers. The angles are measured from the center of the computational domain, $(x, y) = (8 \text{ cm}, 4 \text{ cm})$, to the positions of the fluid markers as functions of time. The angle is increased as the position of the fluid marker is changed in the clockwise direction. We choose arbitrarily 6 fluid markers around the flow loop for each case. These 6 fluid markers are located at the same position initially in all three cases.

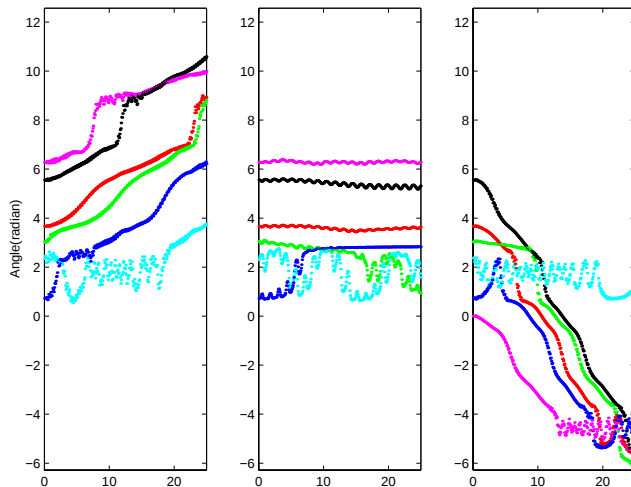


FIG. 7. The angles of fluid markers are plotted as functions of time. The leftmost and rightmost figures show the case of maximum average flow in the clockwise direction (positive slope) and the case of maximum average flow in the counterclockwise direction (negative slope), respectively. The middle figure shows the case of almost zero flow (almost zero slope).

Figure 7 displays the change of the positions of 6 fluid markers by measuring the angle as a function of time for the three cases. The angles are plotted at every 10 time steps up to $t_{max} = 25$ s. Since there are some vortices inside the flexible segment, some fluid markers get trapped and take time to escape the segment. Once fluid markers do escape, however, they move much faster along the rigid part of the racetrack. Examples are the sixth fluid marker in the leftmost frame, the second one in the middle frame, and the third one at the rightmost frame. Some other fluid markers stick to the immersed boundary. Ignoring these details and looking at the general trend, we can see that there is net clockwise motion of the markers in the leftmost frame, no net motion in the middle frame, and net counterclockwise motion in the rightmost frame of Figure 7.

Flowmeter. Figure 8 displays flowmeter results, which are the fluxes computed on the vertical cross section through the middle of the straight segment of tubing at the top of the racetrack. These results are plotted as functions of time over the last 5 cycles in each case. The positive values denote clockwise flow and the negative denote counterclockwise flow. In all three cases the flow is oscillatory, and the oscillation has settled down to a periodic steady state. The flow changes direction with a positive phase and a negative phase during each cycle. In two of the three cases (top and bottom in Figure 8) there is a nonzero mean flow superimposed upon the oscillatory motion. This nonzero mean flow is the phenomenon of valveless pumping.

The motions of wave along the flexible segment. We observe another interesting phenomenon of valveless pumping by investigating the wave motions along the flexible boundary. Figure 9 displays the motions of wave along the top of the flexible segment. Sixteen equal-time snapshots of the wave motions along the top of flexible segment over one cycle of the periodic steady-state are plotted for the three cases. In the top frame, there is a standing wave pattern with two nodes (at about 6.3 cm and 9.8 cm). For reasons that we do not understand, this standing wave

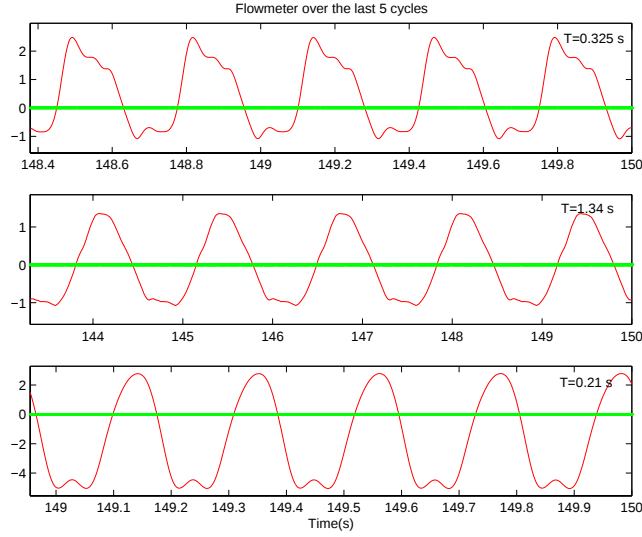


FIG. 8. *Flowmeter.* The fluxes are computed on the vertical cross section through the middle of the straight segment of tubing at the top of the racetrack. They are plotted as functions of time are plotted over the last five cycles in each case (note the different time scales). The case of maximum average flow in the clockwise direction, almost zero flow, and maximum flow in the counterclockwise direction are considered from top and bottom. Note that the mean flow is positive (clockwise) in the top frame and negative (counterclockwise) in the bottom frame.

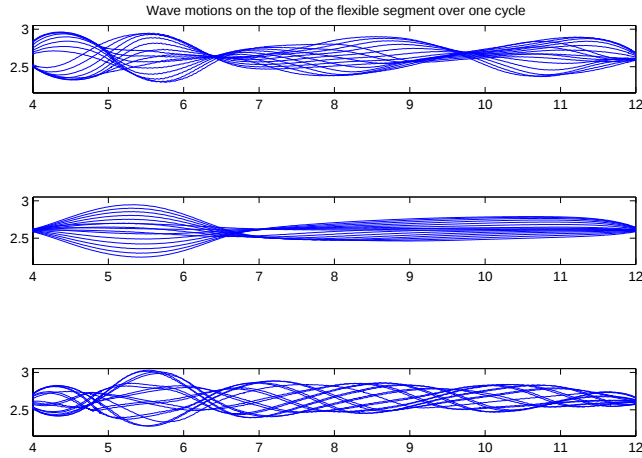


FIG. 9. Sixteen equal-time snapshots over one cycle of the periodic steady-state wave motions along the top of the flexible segment are plotted. The top frame shows the case of maximum average flow in the clockwise direction. The middle frame shows the case of almost zero net flow. The bottom frame shows the case of maximum average flow in the counterclockwise direction. In all these cases, the source of vibration is confined to the left 1/3 of the flexible segment, i.e., to the interval from 4 cm to 6.7 cm.

pattern is associated with maximum clockwise flow. In the middle frame, there again seems to be a standing wave pattern with just one node (at about 6.7 cm). Note,

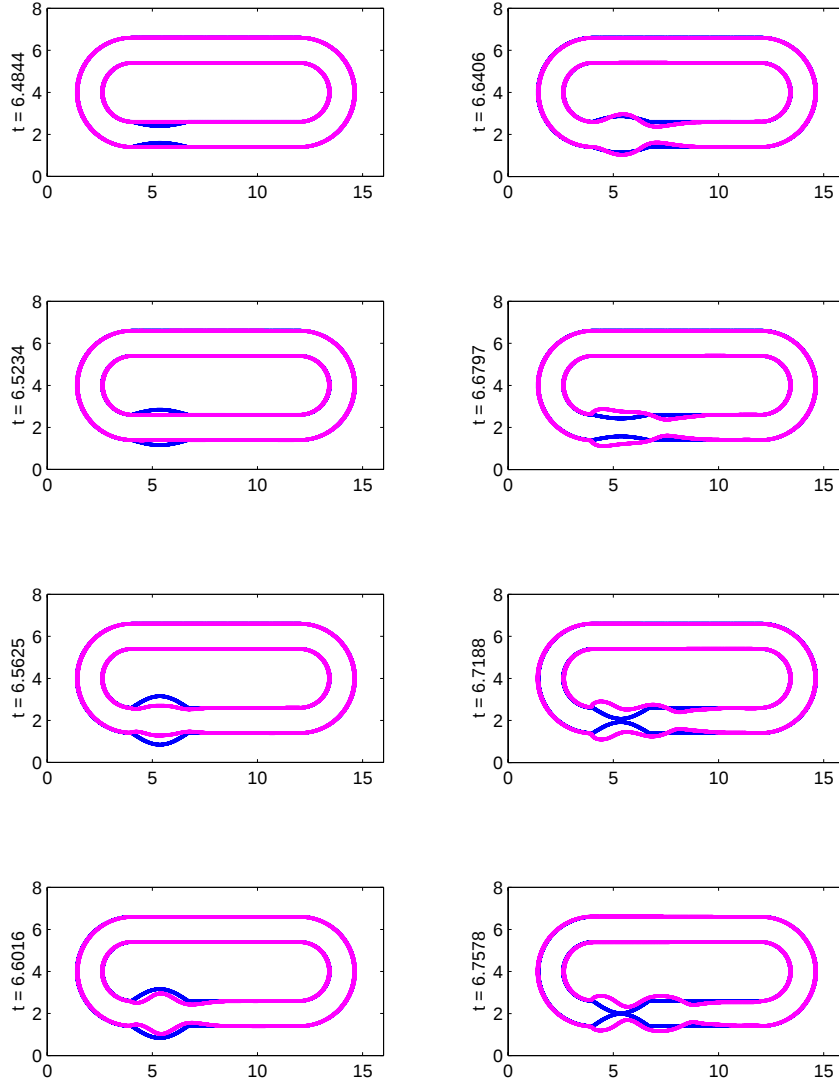


FIG. 10. Comparison of the motions of the target positions (dark) and the physical boundary (light) for the case of maximum average clockwise flow.

however, that the location of this node coincides with the edge of the driven part of the flexible boundary, i.e., the part where an oscillation of the target positions is imposed. Thus, it seems that the driven part of the flexible boundary is oscillating in one phase, and that the rest of the flexible boundary is oscillating in antiphase with flexible part. This wave pattern seems to be associated with the absence of valveless pumping, i.e., with zero net flow. In the bottom frame we see traveling waves (note the absence of nodes) propagating to the right, away from the driven part of the flexible boundary. Such traveling waves might be expected to pump fluid in the direction of propagation by a peristaltic mechanism, and indeed what we see in this case is maximum counterclockwise flow.

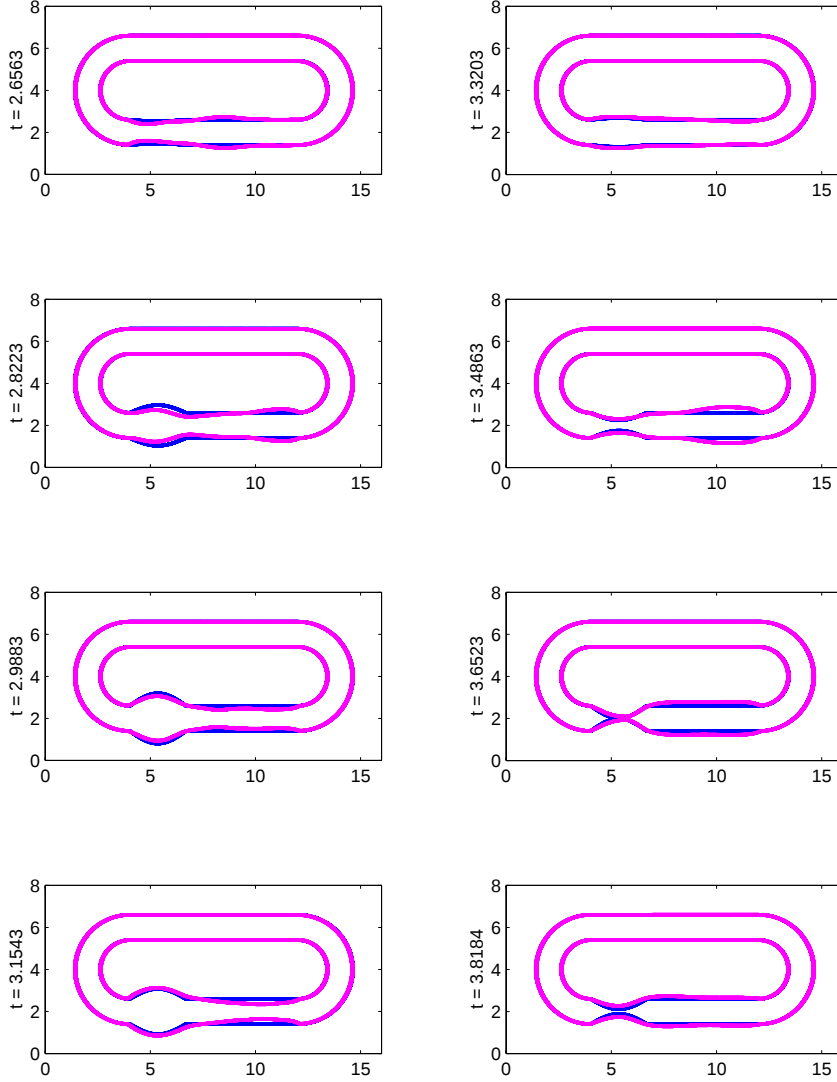


FIG. 11. *Comparison of the motions of the target positions (dark) and the physical boundary (light) for the case of almost zero flow.*

Comparison of the motions of the target positions and the real physical boundary. Here we compare the motions of the target positions and the physical boundary for the three cases. This is done in Figure 10 for the case of maximum average clockwise flow, in Figure 11 for the case of almost zero net flow, and in Figure 12 for the case of maximum average counterclockwise flow. The target positions (dark) are the same in all three figures, since the target motion is specified in advance and differs in three cases only with respect to time scale. Note also that the target positions are time dependent only in the left $1/3$ of the flexible segment, which form the bottom of the racetrack.

Only in the case of zero net flow (Figure 11) does the physical boundary motion

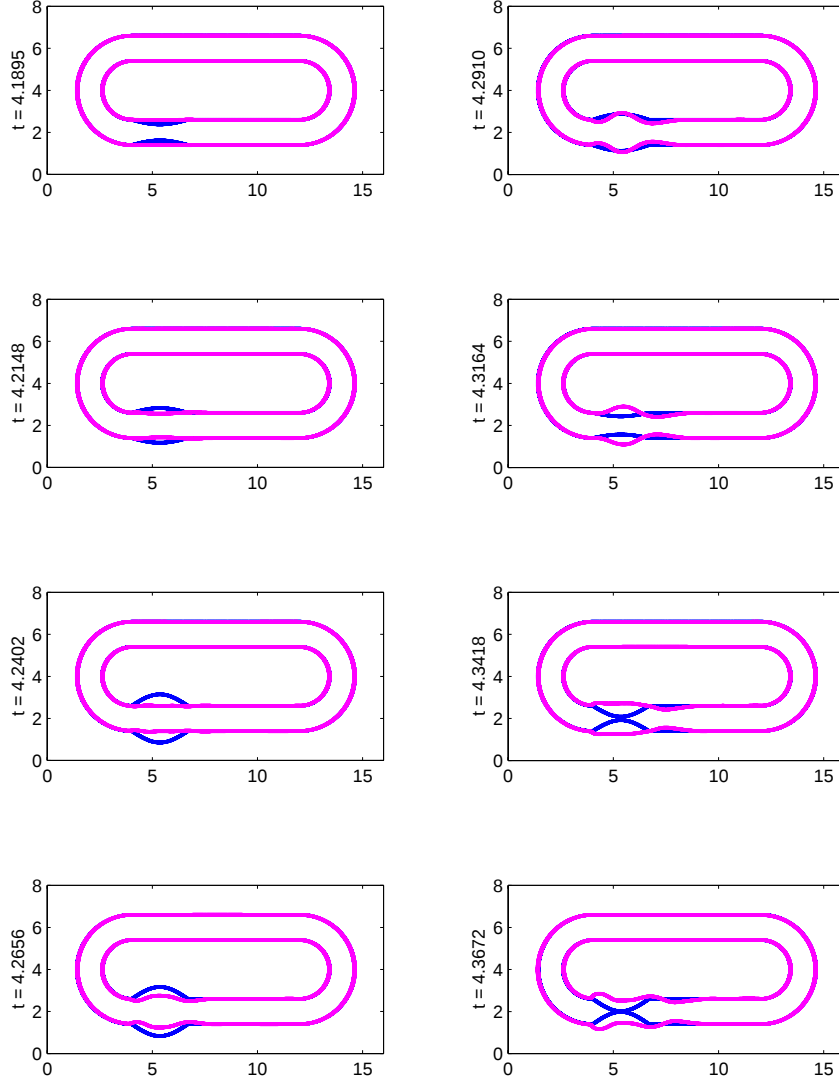


FIG. 12. Comparison of the motions of the target positions (dark) and the physical boundary (light) for the case of maximum average counterclockwise flow.

track the target position motion. This is probably because the frequency is too low for inertia to introduce any phase lags. In the other two cases, there are substantial phase differences (presumably consequences of fluid inertia) between the target position and the physical boundary position.

The velocity vector fields and pressure contours. Figures 13 and 14 display the velocity vector fields of the maximum clockwise flow (period = 0.325 s). Four equal-time snapshots over one cycle of the periodic steady-state are plotted. Figures 15 and 16 display the velocity vector fields of the maximum counterclockwise flow (period = 0.21 s). Four equal-time snapshots over one cycle of the periodic steady-state are also plotted.

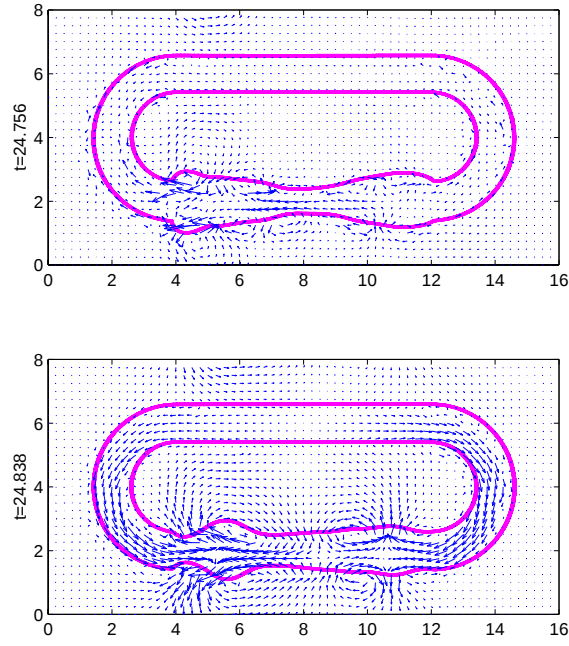


FIG. 13. Velocity vector fields of the maximum clockwise flow. Four equal-time snapshots over one cycle of the periodic steady-state are plotted here and in Figure 14.

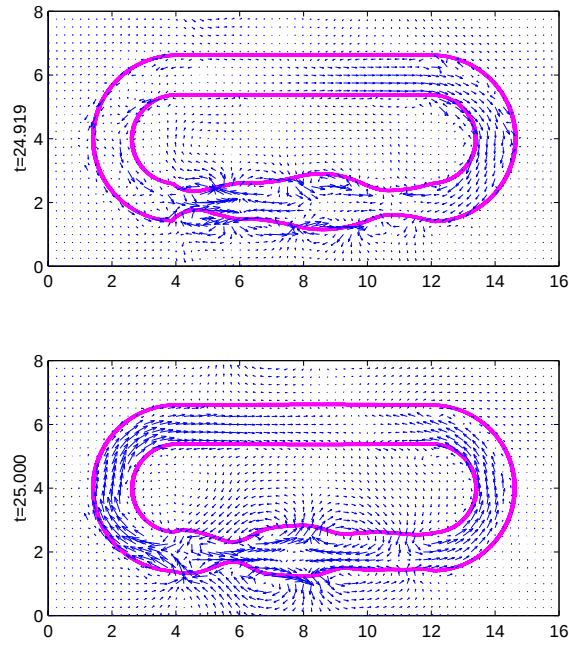


FIG. 14. Continuation of Figure 13.

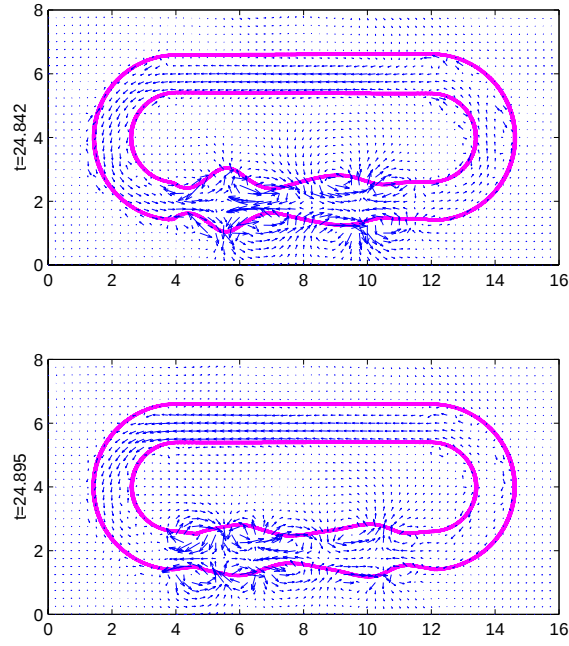


FIG. 15. Velocity vector fields of the maximum counterclockwise flow. Four equal-time snapshots over one cycle of the periodic steady-state are plotted here and in Figure 16.

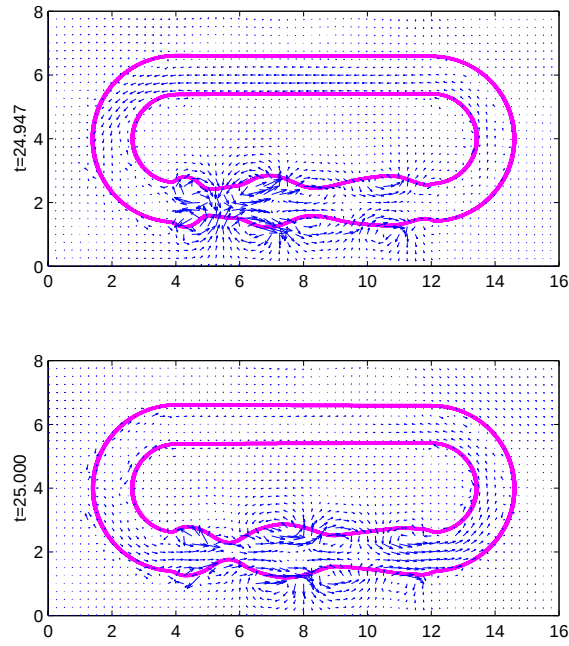


FIG. 16. Continuation of Figure 15.

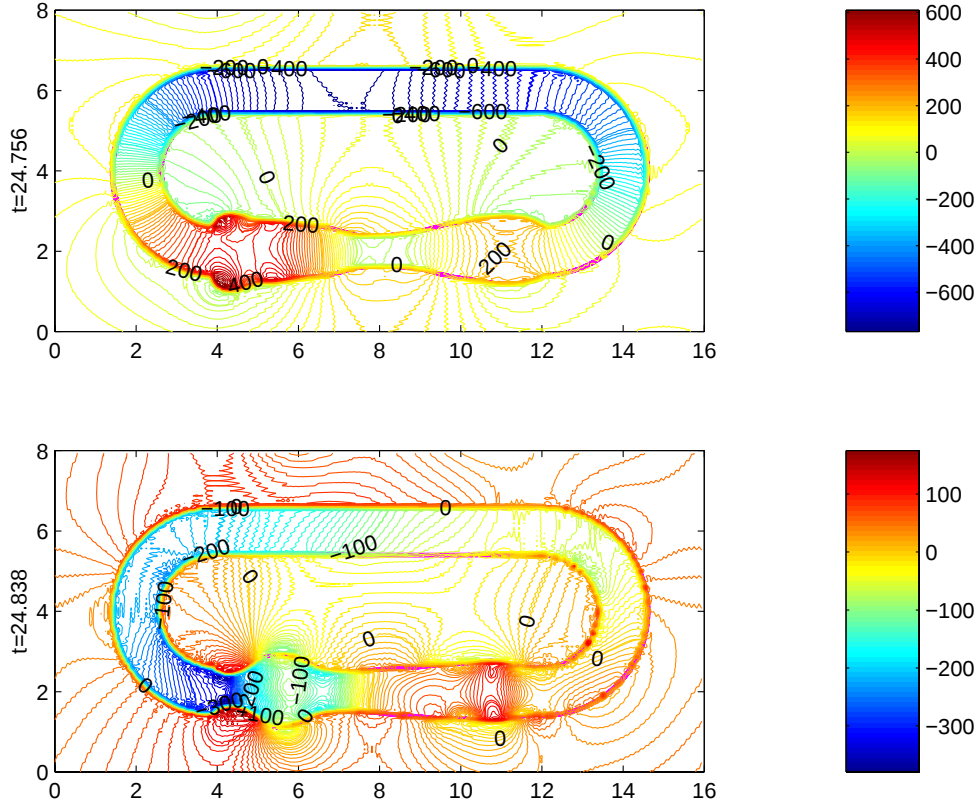


FIG. 17. Pressure contours of the maximum clockwise flow. Four equal-time snapshots over one cycle after the periodic steady-state, plotted here and in Figure 18. The units of pressure are dynes/cm².

Figures 17 and 18 display the pressure contours of the maximum clockwise flow. Four equal-time snapshots over one cycle of the periodic steady-state are plotted. Figures 19 and 20 display the pressure contours of the maximum counterclockwise flow. Four equal-time snapshots over one cycle of the periodic steady-state are also plotted.

Note that it is true that the positions of the immersed boundary are influenced not only by the fluid inside of the loop but also by the fluid outside of the loop. However, motions of the fluid inside the loop seem to be dominant, since that is where the larger velocities and pressure gradients are typically seen in Figures 13–20.

5.2. Further case studies. Flow which is changing the direction by changing the period *during* a computer experiment (periods, $T = 0.325$ s and $T = 0.21$ s). In this section, we present two more interesting cases. In Figure 21, we show that the result does not depend on initial conditions. One might worry that once the flow starts going one way it will keep going that way, but this case shows this is *not* true. In order to show that the crucial parameter to decide the direction of a net flow is frequency, we have examined the situation in which the period of the driving oscillation is 0.325 s for the first half of the simulated experiment, and then

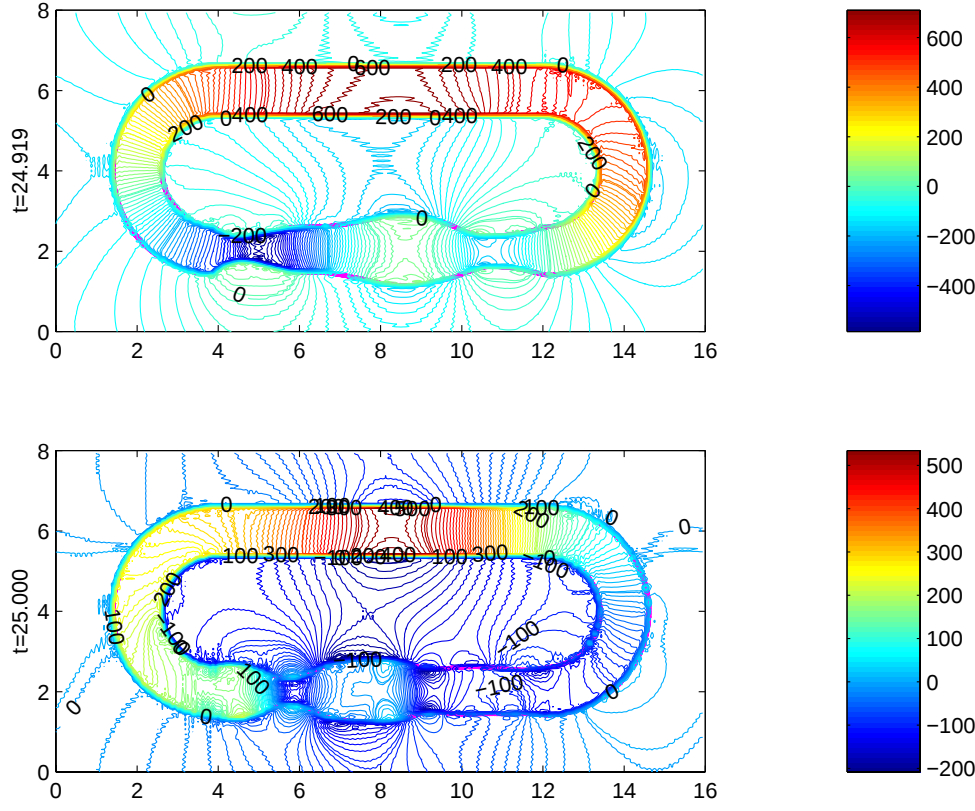


FIG. 18. Continuation of Figure 17.

changes to 0.21 s for the balance of the simulated experiment. Note that these are the periods which generated maximum net flow in the clockwise and counterclockwise direction, respectively. All other parameters except the period are fixed during the simulation.

Figure 21 displays the changing of the positions of two arbitrary fluid markers inside the flow loop by measuring the angles from the center, $(x, y) = (8 \text{ cm}, 4 \text{ cm})$, to the current positions of the markers. We plot angles as functions of time at every 10 time steps up to $t = 20 \text{ s}$ for each fluid marker. The curves in Figure 21 are changing from increasing (clockwise direction) to decreasing (counterclockwise direction) around 10 s.

Zero net flux for the symmetric driving force. Is there still a net flow if the system of valveless pumping would be symmetric? We consider the following special case to show that there is almost zero net flow when the periodic driving forcing is imposed on the center of the flexible segments. All other parameters are chosen for the case of the maximum counterclockwise flow, $T = 0.21 \text{ s}$. This experiment is run until the periodic steady-state $t = 150 \text{ s}$. The time-averaged flux for this case is $-0.023625 \text{ cm}^2/\text{s}$. This shows that there is almost zero net flow when the system is symmetric.

From this case and the previous one, we see that valveless pumping does *not* represent an instability of a symmetric situation. On the contrary, the direction of

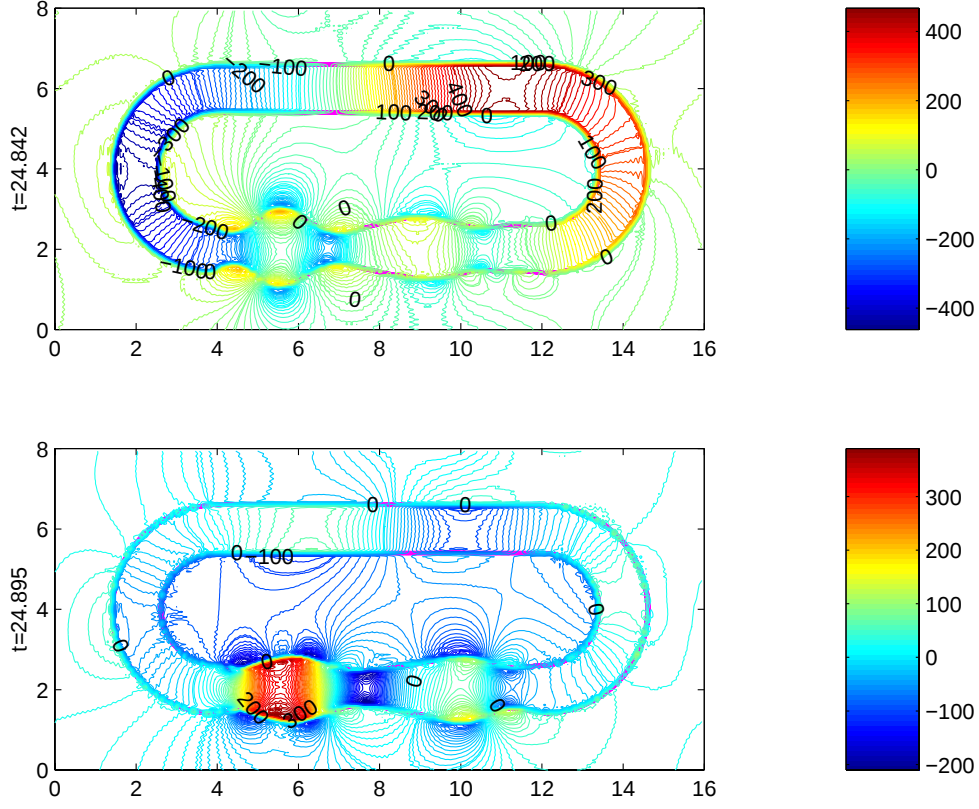


FIG. 19. Pressure contours of the maximum counterclockwise flow. Four equal-time snapshots over one cycle after the periodic steady-state, plotted here and in Figure 20. The units of pressure are dynes/cm².

the mean flow is determined by the asymmetry of the problem but in a frequency and amplitude dependent manner.

6. Conclusions. We have presented numerical experiments concerning “valveless pumping” in the two-dimensional case using the immersed boundary method. As in the earlier papers and physical experiments of valveless pumping, we have also observed the existence of a net flow. Furthermore, we have presented the new, unexpected result that the direction of the flow around the loop of tubing is decided not only by the position of the driving oscillations but also by the *frequency* and the *amplitude* of the driving oscillations. Since CPR may involve valveless pumping, it is of obvious importance to know what frequency and amplitude of chest compression will produce the most effective CPR. Of course we cannot hope to answer this question quantitatively with such an idealized model, but perhaps we have shown qualitatively what phenomena may be expected as the frequency and amplitude of the driving oscillation are varied. We have put special emphasis on the conditions that generate maximum net flow, since that is the goal of CPR.

In studying these cases, we have found an interesting phenomenon: the clockwise net flow seems to be associated with a standing wave in the flexible segment of tubing,

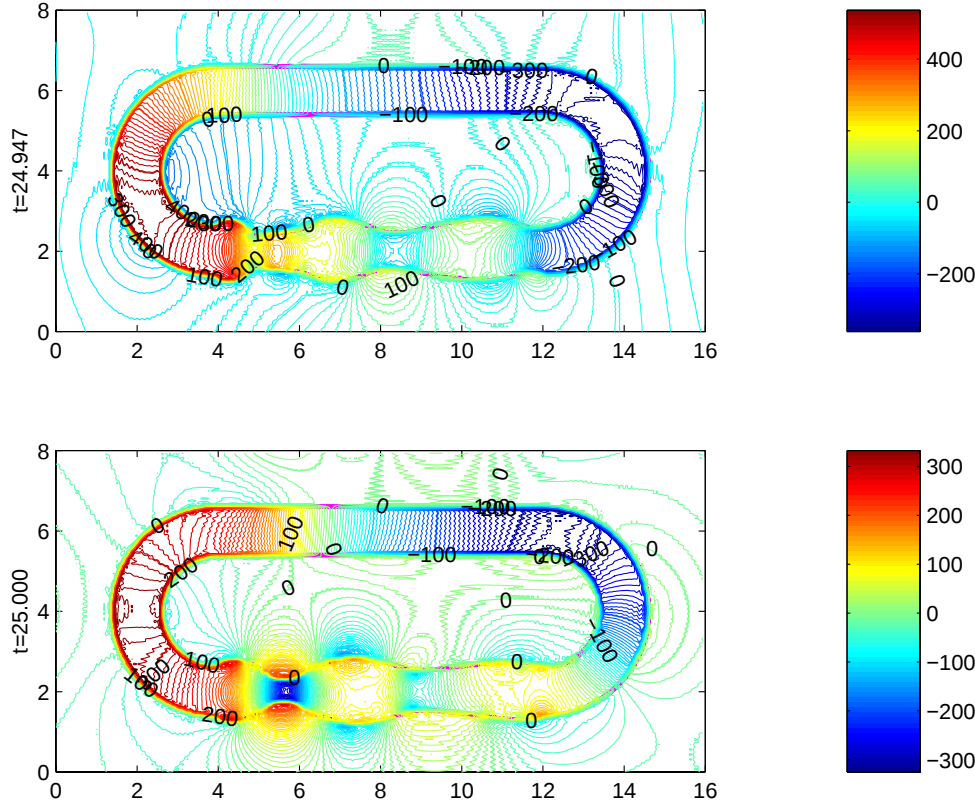


FIG. 20. Continuation of Figure 19.

whereas the counterclockwise net flow seems to be associated with a traveling wave. In the clockwise case (standing wave) the flow in the flexible segment of tubing is going toward the site at which the periodic forcing is applied, but in the counterclockwise case (traveling wave) it is going away from that site and in the same direction as the traveling wave. Therefore, we believe that the counterclockwise flow is driven by the traveling wave via a peristaltic mechanism, but we have no explanation for the clockwise flow in the standing-wave case.

The immersed boundary methodology used here may also be applicable to other biological instances of valveless pumping, such as the blood circulation within the human embryo at the end of the third week of gestation, and to engineering applications such as the design of MEMS.

We are confident that numerical experiments such as those begun in this paper will help answer many questions about the mechanism of valveless pumping. Even though the results demonstrate success in modeling valveless pumping, there is still much future work that remains to be done, such as giving a theoretical explanation for this mysterious phenomenon, and extending this model to the three-dimensional case in order to make it more realistic and more applicable to real-world biomedical problems, like CPR.

A physical experiment of valveless pumping is being constructed at the Courant

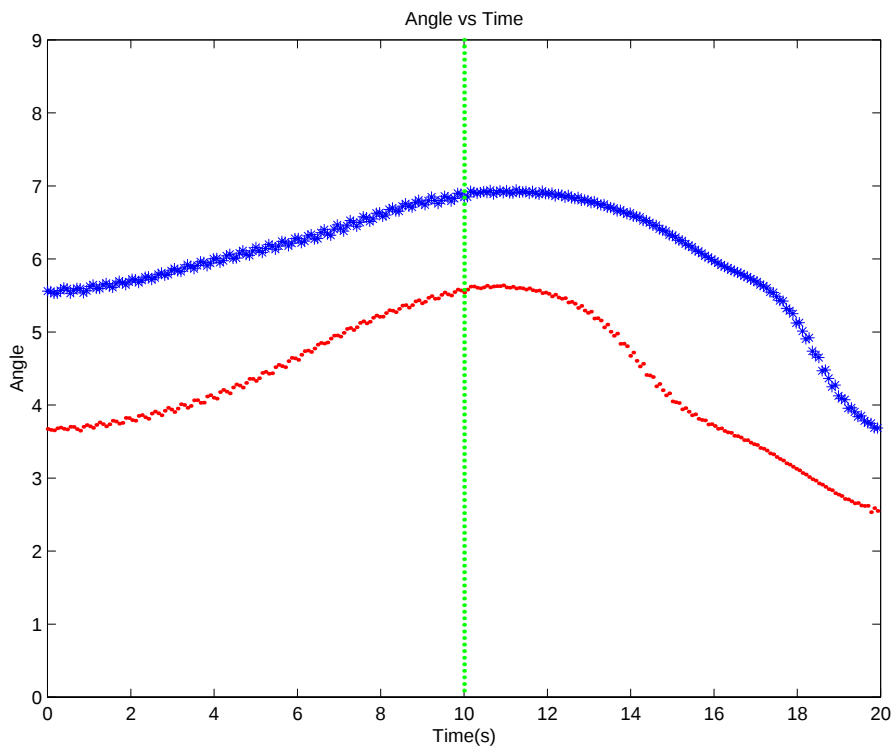


FIG. 21. Flow which is changing the direction by changing the period during a computer experiment: The angles of the positions of two fluid markers are changed from increasing (clockwise) to decreasing (counterclockwise) in time by changing the period from 0.325 s to 0.21 s at time = 10 s. This result shows that the result does not depend on the initial condition.

Institute WetLab. An important part of the future work will be the comparison of computed and experimental results.

Acknowledgments. The authors are especially grateful to David M. McQueen for many helpful discussions and to Jun Zhang for making the physical experiment of valveless pumping. Thanks also to Simcha Milo for pointing out the problem, to Philip Kilner for the experiments that inspired this research project and for helpful discussions, and to Mory Gharib for further experimental insight into valveless pumping.

REFERENCES

- [1] C. BEATTIE, A. D. GUERCI, T. HALL, A. M. BORKON, W. BAUMGARTNER, R. S. STUART, J. PETERS, H. HALPERIN, AND J. L. ROBOTHAM, *Mechanisms of blood flow during pneumatic vest cardiopulmonary resuscitation*, J. Appl. Physiol., 70 (1991), pp. 454–465.
- [2] R. P. BEYER, *A computational model of the cochlea using the immersed boundary method*, J. Comput. Phys., 98 (1992), pp. 145–162.
- [3] J. M. CRILEY, J. T. NIEMANN, J. P. ROSBOROUGH, S. UNG, AND J. SUZUKI, *The heart is a conduit in CPR*, Crit. Care Med., 9 (1981), p. 373.
- [4] M. P. FENELEY, G. M. MAIER, J. W. GAYNOR, S. G. GALL, J. K. KISSLO, J. W. DAVIS, AND J. S. RANKIN, *Sequence of mitral valve motion and transmitral blood flow during manual cardiopulmonary resuscitation in dogs*, Circulation, 76 (1987), pp. 363–375.

- [5] L. J. FAUCI AND C. S. PESKIN, *A computational model of aquatic animal locomotion*, J. Comput. Phys., 77 (1988), pp. 85–108.
- [6] L. J. FAUCI, *Peristaltic pumping of solid particles*, Comput. & Fluids, 21 (1992), pp. 583–598.
- [7] A. L. FOGELSON, *A mathematical model and numerical method for studying platelet adhesion and aggregation during blood clotting*, J. Comput. Phys., 56 (1984), pp. 111–134.
- [8] A. L. FOGELSON AND C. S. PESKIN, *A fast numerical method for solving the three-dimensional Stoke's equations in the presence of suspended particles*, J. Comput. Phys., 79 (1988), pp. 50–69.
- [9] S. GREENBERG, D. M. MCQUEEN, AND C. S. PESKIN, *Three-dimensional fluid dynamics in a two-dimensional amount of central memory*, in Wave Motion: Theory, Modelling, and Computation, Math. Sci. Res. Inst. Publ. 7, Springer-Verlag, New York, 1987, pp. 85–146.
- [10] H. R. HALPERIN, J. E. TSITLIK, R. BEYAR, N. CHANDRA, AND A. D. GUERCI, *Intrathoracic pressure fluctuations move blood during CPR: Comparison of hemodynamic data with predictions from a mathematical model*, Ann. Biomed. Engrg., 15 (1987), pp. 385–403.
- [11] E. JUNG, *2-D Simulations of Valveless Pumping Using the Immersed Boundary Method*, Ph.D. thesis, Courant Institute of Mathematical Sciences, New York University, New York, 1999.
- [12] P. J. KILNER, *Formed flow, fluid oscillation and the heart as a morphodynamic pump (abstract)*, European Surgical Research, 19 (1987), pp. 89–90.
- [13] G. LIEBAU, *Die Bedeutung der Tragheitskräfte für die Dynamik des Blutkreislaufs*, Zs Kreislaufforschung, 46 (1957), pp. 428–438.
- [14] G. LIEBAU, *Die Stromungsprinzipien des Herzens*, Zs Kreislaufforschung, 44 (1955), pp. 677–684.
- [15] G. LIEBAU, *Über ein Ventillosoes Pumpprinzip*, Naturwissenschaften, 41 (1954), pp. 327–328.
- [16] D. M. MCQUEEN, C. S. PESKIN, AND E. L. YELLIN, *Fluid dynamics of the mitral valve: Physiological aspects of a mathematical model*, Amer. J. Physiol., 242 (1982), pp. H1095–H1110.
- [17] S. MIYAZAKI, T. KAWAI, AND M. ARARAGI, *A piezo-electric pump driven by a flexural progressive wave*, in Proceedings of the IEEE Micro Electro Mechanical Systems Workshop, Nara, Japan, 1991, pp. 283–288.
- [18] M. MOSER, J. W. HUANG, G. S. SCHWARZ, T. KENNER, AND A. NOORDERGRAAF, *Impedance defined flow, generalisation of William Harvey's concept of the circulation—370 years later*, Internat. J. Cardiovascular Medicine and Science, 1 (1998), pp. 205–211.
- [19] C. S. PESKIN, *Flow Patterns Around Heart Valves: A Digital Computer Method for Solving the Equations of Motion*, Ph.D. thesis, Albert Einstein College of Medicine, New York, 1972.
- [20] C. S. PESKIN, *Numerical analysis of blood flow in the heart*, J. Comput. Phys., 25 (1977), pp. 220–252.
- [21] C. S. PESKIN AND D. M. MCQUEEN, *A three-dimensional computational method for blood flow in the heart: Immersed elastic fibers in a viscous incompressible fluid*, J. Comput. Phys., 81 (1989), pp. 372–405.
- [22] C. S. PESKIN AND D. M. MCQUEEN, *A general method for the computer simulation of biological systems interacting with fluids*, Symposia of the Society for Experimental Biology, 49 (1995), pp. 265–276.
- [23] C. S. PESKIN AND D. M. MCQUEEN, *Fluid dynamics of the heart and its valves*, in Case Studies in Mathematical Modeling—Ecology, Physiology, and Cell Biology, 1996, pp. 309–337.
- [24] C. S. PESKIN AND B. F. PRINTZ, *Improved volume conservation in the computation of flows with immersed elastic boundaries*, J. Comput. Phys., 105 (1993), pp. 33–46.
- [25] M. T. RUDIKOFF, W. L. MAUGHAN, M. EFFRON, P. FREUND, AND M. L. WEISFELDT, *Mechanisms of blood flow during cardiopulmonary resuscitation*, Circulation, 61 (1980), p. 345.
- [26] H. THOMANN, *A simple pumping mechanism in a valveless tube*, J. Appl. Math. Phys., 29 (1978), pp. 169–177.
- [27] J. A. WERNER, H. L. GREENE, C. L. JANKO, AND L. A. COBB, *Visualization of cardiac valve motion in man during external chest compression using two-dimensional echocardiography: Implications regarding the mechanism of blood flow*, Circulation, 63 (1981), pp. 1417–1421.