A) (due 02 February, 2 pages max) Discuss the solution to the first-order PDE for the problem GL 2-1 #5 (p 24). By discussion, I mean use the solution of this problem to illustrate the ideas presented in the lectures. Use Matlab to plot characteristics on an \( x, t \)-diagram. How might one plot the solution \( u \)?

Use the \texttt{subplot} command to put several plots on a page. All plots MUST be annotated (handwritten is fine) by way of explanation for each plot: highlight salient features, give parameters, etc.

B) (aim for 3 pages) Consider the PDE problem for \( u(x, t) \) for forward time \( (t \geq X) \)

\[
    u_t + cu_x = 0 \quad ; \quad u(-vX, X) = \sin X \quad \text{for} \quad -\infty < X < +\infty
\]

where \( c, v \) are constants. Set up the characteristic ODEs, and be very careful about their initialization (IV) at \( T = 0 \) — this should now allow convenient expressions of the solution as \( U(X, T) \) and \( u(x, t) \). Discuss the \( x, t \)-diagram — are there different cases depending on the sizes and signs of \( c, v \)?

Take \( c, v \) both positive. Describe the solution at a fixed value of \( x \). Likewise, for fixed \( t \). Do the observed periodicities make sense? What is so special about the line \( x + vt = 0 \) in the \( x, t \)-diagram, and why is this sometimes called a moving source problem?

C) (3 pages max) The classic nonlinear, first-order PDE is the IVP

\[
    u_t + uu_x = 0 \quad ; \quad u(X, 0) = f(X)
\]

which is also CP 6.4.2 (p 97). Discuss the formulas and produce illustrative Matlab plots of the characteristics that correspond to 2 choices of IVs: \( f(X) = \pm \tanh X \).

For instance, given any \( (x, t) \) what are the corresponding values of \( (X, T) \)? Computationally estimate at what time the solution ceases to be meaningful (Challenge question: this time can also be directly calculated from the formulas!).