HW7 - Due 03/26/2008
ODE - spring 2008

1) If \( \phi_1, \phi_2, ..., \phi_n \) is a fundamental set for the homogeneous equation
\[
L(x) = x^{(n)} + a_1x^{(n-1)} + ... + a_n x = 0
\]
where \( a_1, a_2, ..., a_n \in C(I) \) are continuous functions of \( t \), then find the solution of \( L\psi = b(t) \) where \( b \in C(I) \) and 
\[
\psi(\tau) = \psi'(\tau) = \psi^{(n-1)}(\tau) = 0, \quad \tau \in I.
\]

2) Determine the stability property of the solution \((sin(t), cos(t))\) of the system
\[
\begin{align*}
x' &= +y(x^2 + y^2)^{-1/2} \\
y' &= -x(x^2 + y^2)^{-1/2}
\end{align*}
\]

3) Prove that the equation
\[
x'' + 2\mu x' + \omega_0^2 x = sin(\omega t)
\]
where \( 0 < \mu < \omega_0 \) and \( \omega > 0 \) has a periodic solution and study its stability.

4) Consider the second order equation
\[
x'' + x^2 + bx + c = 0
\]
where \( b \) and \( c \) are two parameters.
   a/ Determine the critical points and their stability.
   b/ Give a condition for periodic solutions to exist. Are these solution Lyapunov-stable or orbitally stable or asymptotically stable?