## Name:

## HW6 - Due 03/12/2008 <br> ODE - spring 2008

1) Find $x$ such that

$$
\begin{equation*}
x^{(3)}-x^{(2)}+4 x^{\prime}-4 x=0 \tag{1}
\end{equation*}
$$

and $x(0)=-x^{\prime}(0)=x^{(2)}=1$.
2) What is the smallest $n>0$ for which there is a differential equation

$$
\begin{equation*}
x^{(n)}+a_{1} x^{(n-1)}+\ldots+a_{n} x=0 \tag{2}
\end{equation*}
$$

having among its solution $\sin 2 t, 4 t^{2} e^{2 t}$ and $-e^{-t}$. Find the constant $a_{1}, \ldots a_{n}$.
3) Find a real valued function $x$ such that

$$
\begin{equation*}
x^{\prime \prime}+4 x=\cos (2 t) \tag{3}
\end{equation*}
$$

and $x(0)=x^{\prime}(0)=1$.
4) Let $q(t)$ be a polynomial of degree $m$. Show that any equation

$$
\begin{equation*}
x^{(n)}+a_{1} x^{(n-1)}+\ldots+a_{n} x=q(t) \tag{4}
\end{equation*}
$$

has a solution which is a polynomial.

